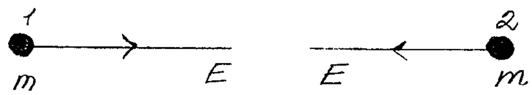
PREPARED BY PIEKAREWICZ
DATE APR 0 5 2002

Homework 10 – Problem 29



In the center-of-momentum frame the four-momentum of each particle is given by:

$$p_1^\mu = \begin{pmatrix} E \\ pc \end{pmatrix} \quad \text{AND} \quad p_2^\mu = \begin{pmatrix} E \\ -pc \end{pmatrix}$$

where the momentum is related to the energy by $(pc)^2 = E^2 - (mc^2)^2$. Equivalently,

$$pc = m\gamma v c = m\gamma c^2\left(\frac{v}{c}\right) = E\beta$$

Thus, to compute the energy of particle 1 in the rest frame of particle 2 we must perform a Lorentz transformation characterized by $\beta \equiv (pc)/E$. That is,

$$\bar{p}_1^\mu \equiv \begin{pmatrix} \bar{E} \\ \bar{p}c \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} E \\ pc \end{pmatrix} = \begin{pmatrix} \gamma E + \beta\gamma pc \\ \beta\gamma E + \gamma pc \end{pmatrix}$$

Hence,

$$\bar{E} = \gamma(E + \beta pc) = \frac{1}{\sqrt{1-\beta^2}}(E + \beta pc) = \frac{E}{\sqrt{E^2 - (pc)^2}}\left(E + \frac{(pc)^2}{E}\right)$$

$$= \left(\frac{E}{mc^2}\right)\left[\frac{E^2 + (E^2 - (mc^2)^2)}{E}\right] = \frac{2E^2}{mc^2} - mc^2$$

OR

$$\boxed{\bar{E} = \frac{2E^2}{mc^2} - mc^2} \longrightarrow \boxed{2\frac{(30)^2}{1} - 1 = 1{,}799\,\text{GeV} \simeq 60E}$$

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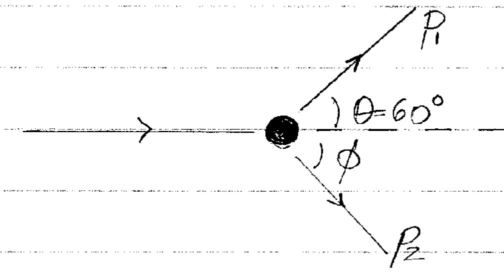
HOMEWORK 10 - PROBLEM 30



WE WRITE THE VARIOUS 4-MOMENTA OF THE PARTICLES (IN UNITS IN WHICH $c=1$)

$$p^{\mu}_{e^-} = (E_e, \vec{p}_e) \,, \quad p^{\mu}_{e^+} = (m, \vec{0})$$

$$p_1^{\mu} = (p_1, \vec{p}_1) \,; \quad p_2^{\mu} = (p_2, \vec{p}_2)$$

NOW USING THE CONSERVATION OF:

ENERGY: $E_e + m = p_1 + p_2$ ①

X-MOMENTUM: $p_e = p_1 \cos\theta + p_2 \cos\phi$ ②

Y-MOMENTUM: $0 = p_1 \sin\theta - p_2 \sin\phi$ ③

FROM ② AND ③ WE OBTAIN:

$$\left.\begin{array}{l} p_2 \cos\phi = p_e - p_1 \cos\theta \\ p_2 \sin\phi = p_1 \sin\theta \end{array}\right\} \Rightarrow \begin{array}{l} p_2^2 = p_e^2 + p_1^2 - 2 p_e p_1 \cos\theta \\ \quad\; = (E_e + m - p_1)^2 \quad [\text{FROM ①}] \end{array}$$

HENCE,

$$p_e^2 + \cancel{p_1^2} - 2 p_e p_1 \cos\theta = E_e^2 + 2mE_e + m^2 - 2(E_e + m) p_1 + \cancel{p_1^2}$$

OR

$$\left[2(E_e + m) - 2 p_e \cos\theta\right] p_1 = E_e^2 - p_e^2 + m^2 + 2mE_e = 2m^2 + 2mE_e$$

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OR

$$\left(E_e + m - p_e \cos\theta\right) p_1 = m\left(E_e + m\right)$$

OR

$$\boxed{p_1 c = E_1 = \frac{mc^2\left(E_e + mc^2\right)}{E_e + mc^2 - p_e c \cos\theta}}$$

FOR $\theta = 60^\circ$, $\cos\theta = 1/2$

$E_e = \left[(p_e c)^2 + (mc^2)^2\right]^{1/2}$

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HOMEWORK 10 – PROBLEM 31

THE MOST IMPORTANT QUANTITY TO CONSIDER IN THIS PROBLEM IS THE "MANDELSTAM" VARIABLE S, WHICH IS DEFINED AS:

$$S \equiv (P_\pi + P_p)^\mu (P_\pi + P_p)_\mu = (P_\pi + P_p)^2$$

THIS VARIABLE IS USEFUL BECAUSE:

i) ENERGY-MOMENTUM CONSERVATION IT HAS THE SAME VALUE BEFORE AND AFTER THE COLLISION; THAT IS, $S_i = (P_\pi + P_p)^2 = (P_K + P_\Sigma)^2 = S_f$

ii) IT IS A RELATIVISTIC INVARIANT; THAT IS, IT HAS THE SAME VALUE IN THE LABORATORY FRAME AS IN THE CENTER OF MOMENTUM FRAME.

CLEARLY, THE MINIMUM VALUE OF "S" IS OBTAINED IN THE CASE IN WHICH THE KAON AND THE SIGMA ARE PRODUCED AT REST IN THE CENTER-OF-MOMENTUM FRAME. IN THIS CASE:

$$S_{min} = S_f^{COM} = (M_K c^2 + M_\Sigma c^2)^2$$

NOW, THIS QUANTITY EVALUATED BEFORE THE COLLISION IN THE LABORATORY FRAME YIELDS,

$$S_i^{LAB} = P_\pi^2 + P_p^2 + 2P_\pi P_p = (M_\pi c^2)^2 + (M_p c^2)^2 + 2E_\pi M_p c^2$$

HENCE, SINCE $S_f^{COM} = S_i^{LAB} \Rightarrow (M_K c^2 + M_\Sigma c^2)^2 = (M_\pi c^2)^2 + (M_p c^2)^2 + 2E_\pi M_p c^2$

OR

$$\boxed{E_\pi^{MIN} = \frac{(M_K c^2 + M_\Sigma c^2)^2 - (M_p c^2)^2 - (M_\pi c^2)^2}{2M_p c^2} = 1{,}143\ \text{MeV}}$$

OR

$$\boxed{P_\pi^{min} c = \sqrt{(E_\pi^{MIN})^2 - (M_\pi c^2)^2} = 1{,}133\ \text{MeV}}$$