

ADVANCED DYNAMICS — PHY-4241/5227

HOMework 11

(March 22, 2003)

Due on Wednesday, April 2, 2003

PROBLEM 32

(Griffiths Problem 12.59)

A particle of mass  $m$  collides elastically with an identical particle at rest. *Classically*, the outgoing trajectories always make an angle of  $90^\circ$ . Calculate this angle *relativistically*, in terms of  $\phi$ , the scattering angle, and  $v$ , the speed, in the center-of-momentum frame.

PROBLEM 33

The electromagnetic field tensor was defined in class as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu ,$$

and all of its six non-vanishing components were evaluated in terms of the electric and magnetic fields. Compute all the components of the **dual** electromagnetic tensor defined by

$$\tilde{F}^{\mu\nu} \equiv G^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} ,$$

in terms of  $\mathbf{E}$  and  $\mathbf{B}$ . Note that  $\epsilon^{\mu\nu\alpha\beta}$  is the four-dimensional generalization of the Levi-Civita symbol; that is,  $\epsilon^{\mu\nu\alpha\beta} = \pm 1$  if  $\mu\nu\alpha\beta$  is an even(odd) permutation of 0123, and zero otherwise.

PROBLEM 34

(Griffiths Problem 12.50)

In the same way that the contraction, or relativistic dot product, of two four vectors is a Lorentz invariant, the contraction of two relativistic tensors is also a Lorentz invariant.

a) Compute the three Lorentz invariants from the contraction of the following tensors:

$$F^{\mu\nu} F_{\mu\nu} , G^{\mu\nu} G_{\mu\nu} , \text{ and } F^{\mu\nu} G_{\mu\nu} ,$$

in terms of the electric and magnetic ( $\mathbf{E}$  and  $\mathbf{B}$ ) fields.

b) Suppose that in one inertial frame  $\mathbf{B} = 0$  but  $\mathbf{E} \neq 0$  (at some point  $P$ ). Is it possible to find another system in which the *electric field* is zero at  $P$ ?