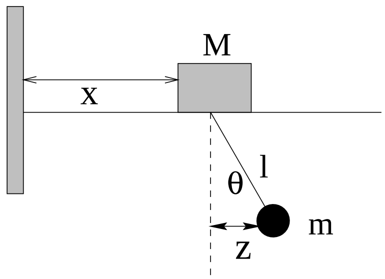
ADVANCED DYNAMICS — PHY–4241/5227

HOMEWORK 12

(April 13, 2003)

Due on Monday, April 21, 2003



PROBLEM 35

(Marion and Thornton Problem 12.3)

Two identical harmonic oscillators (with masses M and natural frequencies ω_0) are coupled such that by adding to the system a mass m common to both oscillators the equations of motions become

$$\ddot{x}_1 + (m/M)\ddot{x}_2 + \omega_0^2 x_1 = 0$$
$$\ddot{x}_2 + (m/M)\ddot{x}_1 + \omega_0^2 x_2 = 0\ .$$

Solve these coupled equations, and obtain the frequencies **and normal modes** of the system.

PROBLEM 36

(Marion and Thornton Problem 12.21)

Three oscillators of equal mass m are coupled such that the potential energy of the system is

$$U = \frac{1}{2}\Big[\kappa_1(x_1^2 + x_3^2) + \kappa_2 x_2^2 + \kappa_3(x_1x_2 + x_2x_3)\Big]\ ,$$

where $\kappa_3 = \sqrt{2\kappa_1\kappa_2}$. Find the eigenfrequencies **and eigenvectors** by solving the secular equation. What is the physical interpretation of the zero-frequency mode?

PROBLEM 37

(similar to Marion and Thornton Problem 12.18)

A simple plane pendulum of mass m and length l hangs from a block of mass M which is constrained to slide on a smooth horizontal track as shown in the figure.

a) Assuming that $\theta \ll 1$, obtain the Lagrangian of the system in the limit of small oscillations. That is, compute the 2×2 symmetric matrices $\hat{\mathcal{T}}$ and $\hat{\mathcal{V}}$ such that the Lagrangian may be written as

$$L = \frac{1}{2}\dot{\eta}^T\hat{\mathcal{T}}\dot{\eta} - \frac{1}{2}\eta^T\hat{\mathcal{V}}\eta\ , \quad \Big(\eta^T \equiv (x, z)\Big)\ .$$

b) Find the normal modes and the associated frequencies of small oscillations of the system.

c) Given the following set of initial conditions: $z(0) = z_0$ and $x(0) = \dot{x}(0) = \dot{z}(0) = 0$, predict the time-dependent motion of the system. That is, compute $x(t)$ and $z(t)$ for all times.