# HOMEWORK 2 – PROBLEM 4

a) THE LAGRANGIAN OF THE SYSTEM IS GIVEN BY

$$L(x,\dot{x}) = T(\dot{x}) - V(x) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2 - \frac{\lambda}{4}x^4$$

TO OBTAIN THE EQUATION OF MOTION WE PERFORM THE FOLLOWING DERIVATIVES:

$$\frac{\partial L}{\partial \dot{x}} = \dot{x} \Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \ddot{x} \quad \text{AND} \quad \frac{\partial L}{\partial x} = -x - \lambda x^3$$

HENCE, THE EULER-LAGRANGE EQUATION OF MOTION BECOMES:

$$\boxed{\ddot{x} + x + \lambda x^3 = 0}$$

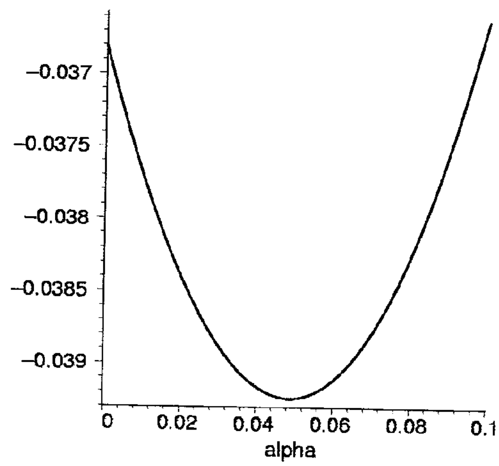
NOTE THAT THIS EQUATION IS A NONLINEAR DIFFERENTIAL EQUATION WHICH I ONLY KNOW HOW TO SOLVE NUMERICALLY. I HAVE DONE THAT AND OBTAINED FOR THE EXACT PATH:

$$\boxed{S = -0.0397}$$

NOW WE HAVE TO CHECK HOW GOOD ARE THE APPROXIMATE PATHS.

***Problem 4:***

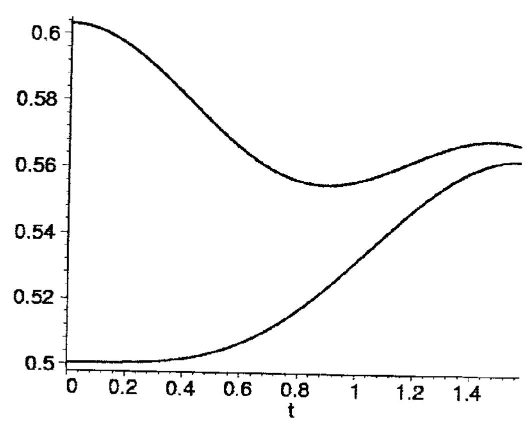
```
> restart:
> lambda := 1/4:
> x0a := sin(t): x1a := diff(x0a,t):
> x0b := sin(t)+alpha*sin(2*t): x1b := diff(x0b,t):
> Ta := x1a^2/2: Va := x0a^2/2+lambda*x0a^4/4: La := Ta-Va:
> Tb := x1b^2/2: Vb := x0b^2/2+lambda*x0b^4/4: Lb := Tb-Vb:
> S0a := int(La,t=0..Pi/2):
> S0b := int(Lb,t=0..Pi/2): S1b := diff(S0b,alpha):
> plot(S0b,alpha=0..0.1,thickness=3);
```




```
> alpha0 := fsolve(S1b,alpha,0..0.5):
> S0b := evalf(subs(alpha=alpha0,S0b)):
> Tb := subs(alpha=alpha0,Tb): Vb := subs(alpha=alpha0,Vb):
> print(alpha0,evalf(S0a),S0b);
```

.04891055266, -.03681553891, -.03925416979

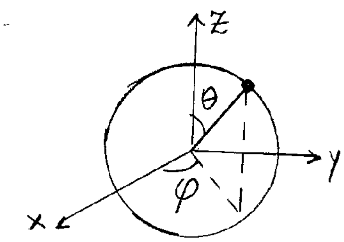
```
> Ea := Ta+Va: Eb := Tb+Vb:
> plot([Ea,Eb],t=0..Pi/2,thickness=3,color=[red,blue]);
>
```



PREPARED BY PIEKAREWICZ
DATE JAN 13 2002

# HOMEWORK 2 - PROBLEM 5

a)




$$x = a\sin\theta\cos\varphi = a\sin\theta\cos(\omega t)$$
$$y = a\sin\theta\sin\varphi = a\sin\theta\sin(\omega t)$$
$$z = a\cos\theta$$

HENCE,

$$\dot{x} = a\cos\theta\cos(\omega t)\dot{\theta} - \omega a\sin\theta\sin(\omega t)$$
$$\dot{y} = a\cos\theta\sin(\omega t)\dot{\theta} + \omega a\sin\theta\cos(\omega t)$$
$$\dot{z} = -a\sin\theta\,\dot{\theta}$$

AND

$$T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) = \frac{1}{2}m\left(a^2\dot{\theta}^2 + a^2\omega^2\sin^2\theta\right)$$

$$V = mgz = mga\cos\theta$$

THUS, THE LAGRANGIAN OF THE SYSTEM BECOMES:

$$\boxed{L = T - V = \frac{1}{2}ma^2(\dot{\theta}^2 + \omega^2\sin^2\theta) - mga\cos\theta}$$

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b)

$$h = \dot{\theta} P_{\dot{\theta}} - L = \dot{\theta}\frac{\partial L}{\partial \dot{\theta}} - L = ma^2\dot{\theta}^2 - \frac{1}{2}ma^2\left(\dot{\theta}^2 + \omega^2\sin^2\theta\right) + mga\cos\theta$$

HENCE,

$$h = \frac{1}{2}ma^2\dot{\theta}^2 - \frac{1}{2}ma^2\omega^2\sin^2\theta + mga\cos\theta$$

NOTE THAT SINCE $\frac{\partial L}{\partial t} = 0$ THEN $h$ = CONSTANT OF THE MOTION. THAT IS,

$$\boxed{\begin{aligned} h &= \frac{1}{2}ma^2\dot{\theta}^2 + V_{EFF}(\theta) \\ V_{EFF}(\theta) &= mga\cos\theta - \frac{1}{2}ma^2\omega^2\sin^2\theta \end{aligned}}$$

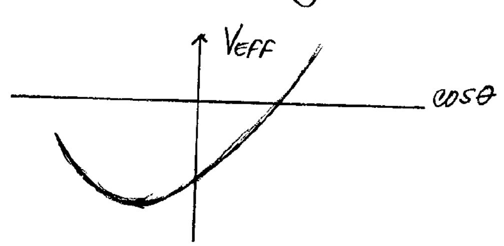
c) FOR THE PARTICLE TO REMAIN STATIONARY AT $\theta \equiv \theta_0 \neq \pi$ WE DEMAND:

$$V'_{EFF}(\theta = \theta_0) = 0 = -mga\sin\theta_0 - ma^2\omega^2\sin\theta_0\cos\theta_0$$

OR

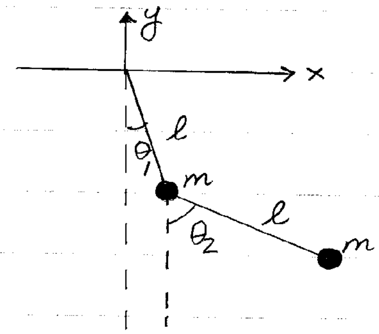
$$\boxed{\cos\theta_0 = -\frac{(g/a)}{\omega^2} \equiv -\frac{\omega_0^2}{\omega^2}}$$

CLEARLY, IF $\omega^2 > \omega_0^2 \equiv g/a$ A PHYSICAL SOLUTION EXISTS FOR $\theta_0 \neq \pi$.



Prepared by: PIEKAREWICZ
Date: JAN 1 8 2003

HOMEWORK 2 - PROBLEM 6




a) TWO PARTICLES MOVING IN THREE DIMENSIONS HAVE SIX DEGREES OF FREEDOM. HOWEVER, THE PLANAR DOUBLE PENDULUM INCLUDES FOUR CONSTRAINTS:

$$Z_1=0\,;\quad Z_2=0\,;\quad X_1^2+Y_1^2=\ell^2\,;\quad (X_2-X_1)^2+(Y_2-Y_1)^2=\ell^2$$

THUS, SIX DEGREES OF FREEDOM PLUS FOUR CONSTRAINTS IMPLIES TWO INDEPENDENT GENERALIZED COORDINATES ($\theta_1$ AND $\theta_2$).

b) IN TERMS OF $\theta_1$ AND $\theta_2$ WE HAVE:

$$X_1=\ell\sin\theta_1\;;\qquad X_2=X_1+\ell\sin\theta_2=\ell\sin\theta_1+\ell\sin\theta_2$$
$$Y_1=-\ell\cos\theta_1\;;\qquad Y_2=Y_1-\ell\cos\theta_2=-\ell\cos\theta_1-\ell\cos\theta_2$$

AND

$$\dot{X}_1=\ell\cos\theta_1\dot{\theta}_1\;;\qquad \dot{X}_2=\ell\cos\theta_1\dot{\theta}_1+\ell\cos\theta_2\dot{\theta}_2$$
$$\dot{Y}_1=\ell\sin\theta_1\dot{\theta}_1\qquad\quad \dot{Y}_2=\ell\sin\theta_1\dot{\theta}_1+\ell\sin\theta_2\dot{\theta}_2$$

THUS,

$$T=\frac{1}{2}m(\dot{X}_1^2+\dot{Y}_1^2)+\frac{1}{2}m(\dot{X}_2^2+\dot{Y}_2^2)$$

$$=\frac{1}{2}m\ell^2\dot{\theta}_1^2+\frac{1}{2}m\left[\ell^2\dot{\theta}_1^2+\ell^2\dot{\theta}_2^2+2\ell^2(\cos\theta_1\cos\theta_2+\sin\theta_1\theta_2)\dot{\theta}_1\dot{\theta}_2\right]$$

$$=\frac{1}{2}m\ell^2\left(2\dot{\theta}_1^2+\dot{\theta}_2^2+2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2\right)$$

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Similarly,

$$V = mgY_1 + mgY_2 = -mgl\cos\theta_1 - mgl\cos\theta_1 - mgl\cos\theta_2$$

$$= -mgl(2\cos\theta_1 + \cos\theta_2)$$

OR

$$\boxed{L = \frac{1}{2}ml^2\left(2\dot\theta_1^2 + \dot\theta_2^2 + 2\cos(\theta_1-\theta_2)\dot\theta_1\dot\theta_2\right) + mgl(2\cos\theta_1 + \cos\theta_2)}$$

c) $$\frac{\partial L}{\partial \dot\theta_1} = 2ml^2\dot\theta_1 + ml^2\cos(\theta_1-\theta_2)\dot\theta_2$$

$$\frac{\partial L}{\partial \dot\theta_2} = ml^2\dot\theta_2 + ml^2\cos(\theta_1-\theta_2)\dot\theta_1$$

$$\frac{\partial L}{\partial \theta_1} = -ml^2\sin(\theta_1-\theta_2)\dot\theta_1\dot\theta_2 - 2mgl\sin\theta_1$$

$$\frac{\partial L}{\partial \theta_2} = -ml^2\sin(\theta_2-\theta_1)\dot\theta_1\dot\theta_2 - mgl\sin\theta_2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} = 2ml^2\ddot\theta_1 + ml^2\cos(\theta_1-\theta_2)\ddot\theta_2 - ml^2\sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\dot\theta_2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} = ml^2\ddot\theta_2 + ml^2\cos(\theta_1-\theta_2)\ddot\theta_1 - ml^2\sin(\theta_1-\theta_2)(\dot\theta_1-\dot\theta_2)\dot\theta_1$$

Thus,

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} - \frac{\partial L}{\partial \theta_1} = 2ml^2\ddot\theta_1 + ml^2\cos(\theta_1-\theta_2)\ddot\theta_2 + ml^2\sin(\theta_1-\theta_2)\dot\theta_2^2 + 2mgl\sin\theta_1$$

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} - \frac{\partial L}{\partial \theta_2} = ml^2\ddot\theta_2 + ml^2\cos(\theta_1-\theta_2)\ddot\theta_1 + ml^2\sin(\theta_2-\theta_1)\dot\theta_1^2 + mgl\sin\theta_2$$

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OR

$$\boxed{\begin{aligned} &2\ddot{\theta}_1 + \cos(\theta_1-\theta_2)\ddot{\theta}_2 + \sin(\theta_1-\theta_2)\dot{\theta}_2^2 + 2\frac{g}{\ell}\sin\theta_1 = 0 \\ &\ddot{\theta}_2 + \cos(\theta_1-\theta_2)\ddot{\theta}_1 + \sin(\theta_2-\theta_1)\dot{\theta}_1^2 + \frac{g}{\ell}\sin\theta_2 = 0 \end{aligned}}$$