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Date: JAN 2 9 2003

HOMEWORK 4 – PROBLEM 10

$$\vec{F} = m\dot{\vec{v}} = \frac{q}{c}\vec{v}\times\vec{B} = \left(\frac{qB_0}{c}\right)\vec{v}\times\hat{z}$$

OR

$$\frac{d\vec{v}}{dt} = \left(\frac{qB_0}{mc}\right)\vec{v}\times\hat{z} = \Omega\left[\hat{x}V_x + \hat{y}V_y + \hat{z}V_z\right]\times\hat{z}$$

$$= \Omega\left(\hat{x}V_y - \hat{y}V_x\right); \quad \Omega \equiv \frac{qB_0}{mc}$$

OR

$$\frac{dV_x}{dt} = \Omega V_y; \quad \frac{dV_y}{dt} = -\Omega V_x; \quad \frac{dV_z}{dt} = 0$$

HENCE,

$$V_z(t) = V_z(0) = \text{CONSTANT}$$

AND

$$\frac{d^2V_x}{dt^2} = \Omega\frac{dV_y}{dt} = \Omega\left[-\Omega V_x\right] \Rightarrow \ddot{V}_x + \Omega^2 V_x = 0$$

THUS,

$$V_x(t) = A\cos\Omega t + B\sin\Omega t$$

AND

$$V_y(t) = \frac{1}{\Omega}\frac{dV_x}{dt} = -A\sin\Omega t + B\cos\Omega t$$

OR

$$\boxed{\begin{aligned} V_x(t) &= V_x(0)\cos\Omega t + V_y(0)\sin\Omega t \\ V_y(t) &= V_y(0)\cos\Omega t - V_x(0)\sin\Omega t \\ V_z(t) &= V_z(0); \quad \Omega \equiv qB_0/mc \end{aligned}}$$

HOMEWORK 4 – PROBLEM 11-12
ADVANCED DYNAMICS – PHY4241

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i)

$$(\nabla\times\vec{A})_i = \varepsilon_{ijk}\partial_j A_k = \varepsilon_{ijk}\partial_j\left[\tfrac{1}{2}\varepsilon_{klm}B_l x_m\right]$$

$$= \tfrac{1}{2}B_l\,\varepsilon_{ijk}\varepsilon_{klm}\delta_{jm} = \tfrac{1}{2}B_l\,\varepsilon_{ijk}\varepsilon_{ljk}$$

$$= \tfrac{1}{2}B_l\, 2\delta_{li} = B_i$$

THUS,

$$\boxed{\vec{A} = \tfrac{1}{2}B_0\hat{z}\times\left[x\hat{x}+y\hat{y}+z\hat{z}\right] = \tfrac{B_0}{2}\left[-y\hat{x}+x\hat{y}\right]}$$

ii)

$$\boxed{L = \tfrac{1}{2}m\dot{\vec{r}}^{\,2} + \tfrac{q}{c}\vec{A}\cdot\vec{v} = \tfrac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right) + \tfrac{qB_0}{2c}\left(-y\dot{x}+x\dot{y}\right)}$$

iii)

$$p_x = \frac{\partial L}{\partial\dot{x}} = m\dot{x} - \frac{qB_0}{2c}y\;;\quad p_y = \frac{\partial L}{\partial\dot{y}} = m\dot{y} + \frac{qB_0}{2c}x\;;\quad p_z = \frac{\partial L}{\partial\dot{z}} = m\dot{z}$$

THEN,

$$H = \sum_i \dot{x}_i p_i - L = \dot{x}\left(m\dot{x}-\alpha y\right) + \dot{y}\left(m\dot{y}+\alpha x\right) + m\dot{z}^2$$

$$-\tfrac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right) - \alpha\left(-y\dot{x}+x\dot{y}\right)$$

$$= \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m\dot{y}^2 + \tfrac{1}{2}m\dot{z}^2$$

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OR

$$H = \tfrac{1}{2}m\left(\frac{P_x+\alpha y}{m}\right)^2 + \tfrac{1}{2}m\left(\frac{P_y-\alpha x}{m}\right)^2 + \tfrac{1}{2}m\left(\frac{P_z}{m}\right)^2$$

OR

$$\boxed{H = \frac{(P_x+\alpha y)^2}{2m} + \frac{(P_y-\alpha x)^2}{2m} + \frac{P_z^2}{2m}\ ; \quad \alpha = \frac{qB_0}{2c}}$$

iv)

FROM ABOVE, WE OBTAIN FOR THE MECHANICAL MOMENTUM:

$$\Pi_x = m\dot{x} = P_x + \alpha y\ ; \quad \Pi_y = m\dot{y} = P_y - \alpha x\ ; \quad \Pi_z = m\dot{z} = P_z$$

THEN,

$$[\Pi_x, \Pi_y] = [P_x+\alpha y, P_y - \alpha x] = \overset{0}{\cancel{[P_x, P_y]}} - \alpha\overset{-1}{\cancel{[P_x, x]}} + \alpha\overset{1}{\cancel{[y, P_y]}} - \alpha^2\overset{0}{\cancel{[y, x]}}$$

OR

$$\boxed{[\Pi_x, \Pi_y] = 2\alpha\ ; \quad [\Pi_y, \Pi_z] = [\Pi_z, \Pi_x] = 0}$$

v)

$$H = \frac{\Pi_x^2}{2m} + \frac{\Pi_y^2}{2m} + \frac{\Pi_z^2}{2m}$$

THEN,

$$\dot{\Pi}_x = [\Pi_x, H] = \frac{1}{2m}[\Pi_x, \Pi_y^2] = \frac{1}{2m}\left\{[\Pi_x,\Pi_y]\Pi_y + \Pi_y[\Pi_x,\Pi_y]\right\} = \frac{2\alpha}{m}\Pi_y$$

$$\dot{\Pi}_y = [\Pi_y, H] = \frac{1}{2m}[\Pi_y, \Pi_x^2] = -\frac{2\alpha}{m}\Pi_x$$

$$\dot{\Pi}_z = [\Pi_z, H] = 0$$

HENCE,

$$\dot{\xi}(t) \equiv \dot{\Pi}_x + i\dot{\Pi}_y = \frac{2\alpha}{m}\left(\Pi_y - i\Pi_x\right) = -\frac{2i\alpha}{m}\left(\Pi_x + i\Pi_y\right) = -\frac{2i\alpha}{m}\xi(t)$$

OR

$$\dot{\xi}(t) = -i\omega_c\,\xi(t) \; ; \qquad \omega_c \equiv \frac{2\alpha}{m} = \frac{qB_0}{mc}$$

AND

$$\dot{\Pi}_z(t) = 0 \Rightarrow \Pi_z(t) = \Pi_z(0) = \text{CONSTANT}$$

HENCE,

$$\xi(t) = \xi(0)\,e^{-i\omega_c t} \equiv \left(\Pi_x(0) + i\Pi_y(0)\right)e^{-i\omega_c t}$$

$$= \left[\Pi_x(0) + i\Pi_y(0)\right]\left[\cos\omega_c t - i\sin\omega_c t\right]$$

OR

$$\boxed{\begin{aligned} \Pi_x(t) &= \Pi_x(0)\cos\omega_c t + \Pi_y(0)\sin\omega_c t \\ \Pi_y(t) &= \Pi_y(0)\cos\omega_c t - \Pi_x(0)\sin\omega_c t \\ \Pi_z(t) &= \Pi_z(0) \; ; \quad \omega_c = qB_0/mc \end{aligned}}$$

FOR A PARTICLE OF CHARGE q MOVING IN THE PRESENCE OF A CONSTANT MAGNETIC FIELD $\vec{B} = B_0\hat{z}$, NEWTON'S LAW + LORENTZ FORCE YIELD:

$$m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{\Pi}}{dt} = \frac{q}{c}\vec{v}\times\vec{B} = \left(\frac{qB_0}{mc}\right)\vec{\Pi}\times\hat{z}$$

THEN, $\vec{\Pi}$ PRECESSES WITH FREQUENCY $\omega_c = \frac{qB_0}{mc}$ ALONG THE $\hat{z}$-AXIS, IN AGREEMENT WITH THE ABOVE RESULT.