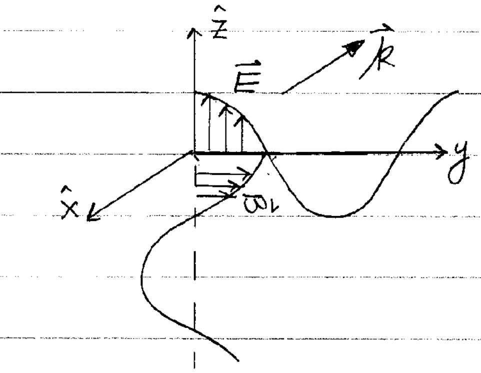
Prepared by:	Piekarewicz
Date:	FEB 09 2003

Homework 6 - Problem 16

a)

$$\vec{E}(\vec{r},t) = \hat{z}E_0 \cos(kx+\omega t) \; ; \quad \text{with } \omega = kc$$

$$\vec{B}(\vec{r},t) = \hat{k}\times\vec{E} = -\hat{x}\times\vec{E} = \hat{y}E_0\cos(kx+\omega t)$$



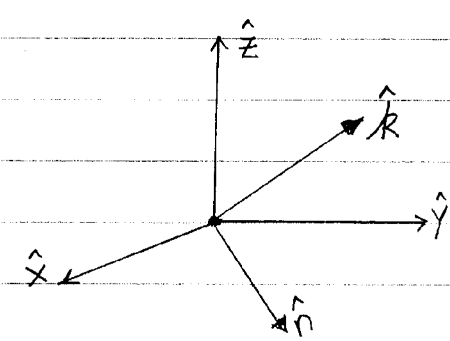
b)

The direction of propagation of the wave is $\hat{k} = \frac{1}{\sqrt{3}}(1,1,1)$.
Two vectors perpendicular to $\hat{k}$ are:

$$\hat{n} = \frac{1}{\sqrt{2}}(1,0,-1) \quad \text{and} \quad \hat{u} = \frac{1}{\sqrt{6}}(-1,2,-1) = \hat{k}\times\hat{n}$$

Hence,

$$\boxed{\begin{aligned}\vec{E}(\vec{r},t) &= \hat{n}E_0\cos(\vec{k}\cdot\vec{r}-\omega t) = \hat{n}E_0\cos\left[\frac{k}{\sqrt{3}}(x+y+z)-\omega t\right]\\ \vec{B}(\vec{r},t) &= \hat{u}E_0\cos(\vec{k}\cdot\vec{r}-\omega t);\ \omega = kc\end{aligned}}$$



It is hard for such a poor artist like me to draw this; but you get the idea.

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HOMEWORK 6 - PROBLEM 17,18

$$B_x(\vec{r},t) = -iB_0 \sin\left(\frac{\pi x}{a}\right) e^{i(kz-\omega t)}$$
$$B_y(\vec{r},t) = 0$$
$$B_z(\vec{r},t) = B_0 \cos\left(\frac{\pi x}{a}\right) e^{i(kz-\omega t)}$$

a)

$$0 = \nabla\cdot\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$
$$= \left[-iB_0 \frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right) + ikB_0 \cos\left(\frac{\pi x}{a}\right)\right] e^{i(kz-\omega t)}$$
$$= -iB_0\left[\frac{\pi}{a} - k\right]\cos\left(\frac{\pi x}{a}\right) e^{i(kz-\omega t)}$$

OR

$$\boxed{k = \pi/a}$$

b)

Assume $\vec{E}(\vec{r},t) = \vec{E}(\vec{r})\, e^{-i\omega t}$

THEN,

$$0 = \nabla\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \nabla\times\vec{B} + \frac{i\omega}{c}\vec{E}$$

OR

$$\vec{E} = i\frac{c}{\omega}(\nabla\times\vec{B}) = i\frac{c}{\omega}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ B_x & 0 & B_z \end{vmatrix} = i\frac{c}{\omega}\hat{y}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)$$

$$= i\frac{c}{\omega}\hat{y}\left\{kB_0 \sin\left(\frac{\pi x}{a}\right) + \frac{\pi}{a}B_0 \sin\left(\frac{\pi x}{a}\right)\right\} e^{i(kz-\omega t)}$$

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OR

$$\boxed{\vec{E}(\vec{r},t) = 2i\left(\frac{kc}{\omega}\right)B_0 \sin\left(\frac{\pi x}{a}\right)e^{i(kz-\omega t)}\,\hat{y}}$$

c)

$$\boxed{\nabla\cdot\vec{E} = \frac{\partial E_y}{\partial y} = 0}$$ since E_y is INDEPENDENT OF y

d)

$$0 = \nabla\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & E_y & 0 \end{vmatrix} - \frac{i\omega}{c}\vec{B}$$

$$= -\hat{x}\frac{\partial E_y}{\partial z} + \hat{z}\frac{\partial E_y}{\partial x} - \frac{i\omega}{c}\vec{B}$$

$$= -\hat{x}(ikE_y) + \hat{z}\left[2i\left(\frac{kc}{\omega}\right)\frac{\pi}{a}B_0\cos\left(\frac{\pi x}{a}\right)e^{i(kz-\omega t)}\right] - \frac{i\omega}{c}\vec{B}$$

$$= 2i\frac{k^2c}{\omega}\left(\hat{x}B_x + \hat{z}B_z\right) - \frac{i\omega}{c}\vec{B}$$

$$= i\left[2\frac{k^2c}{\omega} - \frac{\omega}{c}\right]\vec{B} \Rightarrow 2\frac{k^2c}{\omega} - \frac{\omega}{c} = 0 \Rightarrow \omega^2 = 2(kc)^2$$

OR

$$\boxed{\omega = \sqrt{2}\,kc}$$