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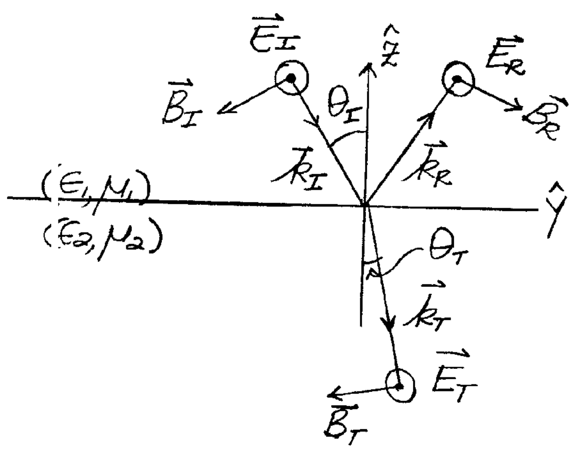
DATE FEB 04 2002

HOMEWORK 7 - PROBLEM 20



a) INCIDENT WAVE:

$$\vec{k}_I = k_I(\sin\theta_I\,\hat{y} - \cos\theta_I\,\hat{z})$$
$$\vec{B}_I = B_I(-\cos\theta_I\,\hat{y} - \sin\theta_I\,\hat{z})\,e^{i(\vec{k}_I\cdot\vec{x}-\omega t)}$$
$$\vec{E}_I = E_I\,\hat{x}\,e^{i(\vec{k}_I\cdot\vec{x}-\omega t)}$$

WITH

$$k_I = \frac{\omega}{c}n_1 = \frac{\omega}{c}\sqrt{\epsilon_1\mu_1} \quad \text{AND} \quad B_I = n_1 E_I$$

REFLECTED WAVE:

$$\vec{k}_R = k_R(\sin\theta_I\,\hat{y} + \cos\theta_I\,\hat{z})$$
$$\vec{B}_R = B_R(\cos\theta_I\,\hat{y} - \sin\theta_I\,\hat{z})\,e^{i(\vec{k}_R\cdot\vec{x}-\omega t)}$$
$$\vec{E}_R = E_R\,\hat{x}\,e^{i(\vec{k}_R\cdot\vec{x}-\omega t)}$$

WITH

$$k_R = \frac{\omega}{c}n_1 = \frac{\omega}{c}\sqrt{\epsilon_1\mu_1} \quad \text{AND} \quad B_R = n_1 E_R$$

TRANSMITTED WAVE:

$$\vec{k}_T = k_T(\sin\theta_T\,\hat{y} - \cos\theta_T\,\hat{z})$$
$$\vec{B}_T = B_T(-\cos\theta_T\,\hat{y} - \sin\theta_T\,\hat{z})\,e^{i(\vec{k}_T\cdot\vec{x}-\omega t)}$$
$$\vec{E}_T = E_T\,\hat{x}\,e^{i(\vec{k}_T\cdot\vec{x}-\omega t)}$$

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WITH

$$k_T = \frac{\omega}{c} n_2 = \frac{\omega}{c}\sqrt{\epsilon_2 \mu_2} \quad \text{AND} \quad B_T = n_2 E_T$$

b) THE FOUR BOUNDARY CONDITIONS, AFTER ALREADY ASSUMING THE EQUALITY OF ALL THE PHASE FACTORS, ARE GIVEN BY:

i) $\hat{z}\cdot\left[\epsilon_1(\vec{E}_I + \vec{E}_R) - \epsilon_2 \vec{E}_T\right] = 0$ (GAUSS' LAW)

ii) $\hat{z}\cdot\left[(\vec{B}_I + \vec{B}_R) - \vec{B}_T\right] = 0$ (NO MAGNETIC MONOPOLES)

iii) $\hat{z}\times\left[(\vec{E}_I + \vec{E}_R) - \vec{E}_T\right] = 0$ (FARADAY'S LAW)

iv) $\hat{z}\times\left[\frac{1}{\mu}(\vec{B}_I + \vec{B}_R) - \frac{1}{\mu_2}\vec{B}_T\right] = 0$ (AMPERE-MAXWELL LAW)

GAUSS' LAW IS TRIVIALLY SATISFIED SINCE THE ELECTRIC FIELDS ARE ALL IN THE $\hat{x}$-DIRECTION. THE OTHER THREE EQUATIONS YIELD:

ii) $0 = \left[-(B_I + B_R)\sin\theta_I + B_T \sin\theta_T\right] = -(E_I + E_R)n_1 \sin\theta_I + E_T n_2 \sin\theta_T$

iii) $0 = \left[(E_I + E_R) - E_T\right]$

iv) $0 = \frac{1}{\mu_1}\left[-\cos\theta_I (B_I - B_R)\right] + \frac{1}{\mu_2}\cos\theta_T B_T$

$= \frac{n_1}{\mu_1}\left[-\cos\theta_I (E_I - E_R)\right] + \frac{n_2}{\mu_2}\cos\theta_T E_T$

OR

$$(E_I + E_R) n_1 \sin\theta_I = E_T n_2 \sin\theta_T \qquad (ii)$$

$$(E_I + E_R) = E_T \qquad (iii)$$

$$\frac{n_1}{\mu_1}(E_I - E_R)\cos\theta_I = \frac{n_2}{\mu_2} E_T \cos\theta_T \qquad (iv)$$

CLEARLY, THE FIRST TWO EQUATIONS ARE IDENTICAL BECAUSE OF SNELL'S LAW. THUS, THE SYSTEM OF EQUATIONS THAT NEED TO BE SOLVED IS GIVEN BY:

$$(E_I + E_R) = E_T$$

$$(E_I - E_R) = \underbrace{\left[\left(\frac{n_2}{n_1}\right)\left(\frac{\mu_1}{\mu_2}\right)\right]}_{\beta}\underbrace{\left(\frac{\cos\theta_T}{\cos\theta_I}\right)}_{\alpha} E_T = \alpha\beta E_T$$

OR

$$\boxed{\begin{aligned} E_I + E_R &= E_T \\ E_I - E_R &= \alpha\beta E_T \end{aligned}} \qquad \begin{aligned} \alpha &\equiv \cos\theta_T / \cos\theta_I \\ \beta &\equiv (n_2\mu_1)/(n_1\mu_2) \end{aligned}$$

c) FROM ABOVE WE OBTAIN

$$(1+\alpha\beta)E_T = 2E_I \Rightarrow E_T/E_I = 2/(1+\alpha\beta)$$

AND

$$\frac{E_R}{E_I} = \frac{E_T}{E_I} - 1 = \frac{2}{1+\alpha\beta} - 1 = (1-\alpha\beta)/(1+\alpha\beta)$$

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OR

$$\boxed{\frac{E_R}{E_I} = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right) \quad ; \quad \frac{E_T}{E_I} = \frac{2}{(1+\alpha\beta)}}$$

THE INTENSITIES MUST BE COMPUTED FROM THE COMPONENT OF THE POYNTING VECTOR PARALLEL TO $\hat{z}$. THAT IS,

$$I \equiv \vec{S}\cdot\hat{z} = \hat{z}\cdot\frac{c}{8\pi}\,\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right) = \hat{z}\cdot\frac{c}{8\pi}\,\mathrm{Re}\left(\vec{E}\times\frac{\vec{B}^*}{\mu}\right)$$

$$= \frac{c}{8\pi}\,\frac{n}{\mu}\cos\theta\, E^2$$

HENCE,

$$R = \frac{I_R}{I_I} = \left(\frac{E_R}{E_I}\right)^2 = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2$$

$$T = \frac{I_T}{I_I} = \frac{(n_2/\mu_2)\cos\theta_T}{(n_1/\mu_1)\cos\theta_I}\left(\frac{E_T}{E_I}\right)^2 = \alpha\beta\,\frac{4}{(1+\alpha\beta)^2}$$

OR

$$\boxed{R = \left(\frac{1-\alpha\beta}{1+\alpha\beta}\right)^2 \quad \text{AND} \quad T = \frac{4\alpha\beta}{(1+\alpha\beta)^2}}$$

WITH

$$R+T = \frac{1-2\alpha\beta+\alpha^2\beta^2+4\alpha\beta}{(1+\alpha\beta)^2} = \frac{1+2\alpha\beta+\alpha^2\beta^2}{(1+\alpha\beta)^2} = 1$$

Problem 14

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> restart:
> mu1:= 1: mu2 := 1: n1 := 1.0: n2 := 1.5:
> sin2 := n1*sin(theta*Pi/180)/n2: cos2 := sqrt(1-sin2^2):
> alpha := cos2/cos(theta*Pi/180): beta := (mu1*n2)/(mu2*n1):
> ERf := (1-alpha*beta)/(1+alpha*beta): ETr := 2/(1+alpha*beta):
> R := ERf^2: T := alpha*beta*ETr^2:
> plot([R,T,R+T],theta=0..90,color=[blue,red,green],thickness=3);
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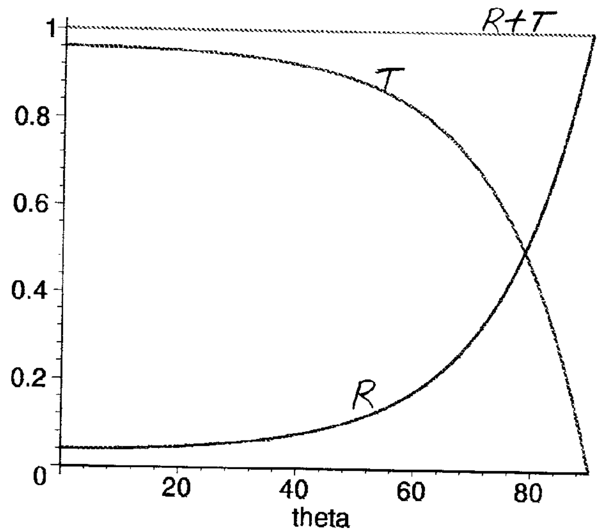

THERE IS NO ANGLE FOR WHICH R=0.

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HOMEWORK 7 – PROBLEM 21

a) WITH $u = \frac{1}{8\pi}\left(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H}\right)$ WE OBTAIN:

$$\frac{\partial u}{\partial t} = \frac{1}{8\pi}\frac{\partial}{\partial t}\left[\frac{\vec{D}}{\epsilon}\cdot\vec{D} + \vec{B}\cdot\frac{\vec{B}}{\mu}\right]$$

$$= \frac{1}{8\pi}\left[\frac{2}{\epsilon}\vec{D}\cdot\frac{\partial\vec{D}}{\partial t} + \frac{2}{\mu}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right]$$

$$= \frac{1}{4\pi}\left[\vec{E}\cdot\left(c\nabla\times\vec{H} - 4\pi\vec{J}\right) + \vec{H}\cdot\left(-c\,\nabla\times\vec{E}\right)\right]$$

$$= -\frac{c}{4\pi}\left[\vec{H}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{H})\right] - \vec{J}\cdot\vec{E}$$

$$= -\nabla\cdot\left[\frac{c}{4\pi}(\vec{E}\times\vec{H})\right] - \vec{J}\cdot\vec{E}$$

OR

$$\boxed{\begin{gathered}\frac{\partial u}{\partial t} + \nabla\cdot\vec{S} = -\vec{J}\cdot\vec{E}\\ \vec{S} :\equiv \frac{c}{4\pi}(\vec{E}\times\vec{H}) = \text{Poynting Vector}\end{gathered}}$$

THE $\vec{J}\cdot\vec{E}$ TERM REPRESENTS A DISSIPATION ("FRICTIONAL") TERM THAT RESULTS WHEN ENERGY STORED IN SPACE IS USED TO DO WORK ON CHARGES. INDEED, THE POWER (WORK PER UNIT TIME) DELIVERED BY THE FIELDS IS GIVEN BY:

$$P = \vec{F}\cdot\vec{v} = \int d^3r\,\rho(\vec{r})\left[\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right]\cdot\vec{v} =$$

$$= \int d^3r\,(\rho(\vec{r})\vec{v})\cdot\vec{E} = \int d^3r\,\vec{J}\cdot\vec{E}$$

b) FOR THIS PART WE WRITE:

$$\vec{E}(\vec{x},t) = \vec{E}_0 \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \equiv \vec{E}(\vec{x}) e^{-i\omega t} \qquad (\vec{k}\cdot\vec{E}=0)$$

$$\vec{B}(\vec{x},t) = \vec{B}_0 \, e^{i(\vec{k}\cdot\vec{x}-\omega t)} \equiv \vec{B}(\vec{x}) e^{-i\omega t} \qquad (\vec{k}\cdot\vec{B}=0)$$

WITH $\vec{B}_0 = n\hat{k}\times\vec{E}_0$

THEN,

$$\vec{S} = \frac{c}{4\pi} Re(\vec{E}) \times Re(\vec{H}) = \frac{c}{4\pi\mu} \left(\frac{\vec{E}+\vec{E}^*}{2}\right) \times \left(\frac{\vec{B}+\vec{B}^*}{2}\right)$$

$$= \frac{c}{16\pi\mu} \left[(\vec{E}\times\vec{B}) + (\vec{E}^*\times\vec{B}^*) + (\vec{E}\times\vec{B}^*) + (\vec{E}^*\times\vec{B}) \right]$$

$$= \frac{c}{8\pi\mu} \left[Re(\vec{E}\times\vec{B}) + Re(\vec{E}\times\vec{B}^*) \right]$$

HOWEVER, THE FIRST TERM TIME-AVERAGES TO ZERO WHILE THE SECOND ONE AVERAGES TO 1. HENCE,

$$\boxed{\langle \vec{S} \rangle_{avg} = \frac{c}{8\pi} Re(\vec{E}\times\vec{H}^*)}$$

FOR THE ABOVE PLANE WAVES:

$$\langle \vec{S} \rangle_{avg} = \frac{c}{8\pi\mu} Re\left[\vec{E}_0 e^{i\vec{k}\cdot\vec{x}} \times \vec{B}_0 e^{-i\vec{k}\cdot\vec{x}} \right] = \frac{c}{8\pi\mu} \left[\vec{E}_0 \times (n\hat{k}\times\vec{E}_0) \right]$$

OR

$$\boxed{\langle \vec{S} \rangle_{avg} = \frac{cn}{8\pi\mu} E_0^2 \hat{k} = \frac{c}{8\pi}(E_0 H_0)\hat{k}}$$

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HOMEWORK 7 - PROBLEM 22

USING NEWTON'S 2nd LAW OF MOTION WE OBTAIN

$$F = eE_0 = m\frac{dv}{dt} \Rightarrow \quad V = \frac{e}{m}E_0 t$$

OR

$$\boxed{t = \frac{mc}{eE_0} \quad \text{OR} \quad ct = \frac{mc^2}{eE_0}}$$

NOTE THAT $mc^2 = .511\times10^6\,eV$ AND $eE_0 = 10^6\,eV/m$. THUS,

$$ct = \frac{.511\times10^6\,eV}{10^6\,eV/m} = .511\,m \quad \text{OR} \quad t = \frac{.511\,m}{3\times10^8\,m/s} = .17\times10^{-8}\,s$$

OR

$$\boxed{t = 1.7\times10^{-9}\,s}$$

IT WOULD TAKE THE ELECTRON 1.7 NANOSECONDS TO REACH THE SPEED OF LIGHT.

NOTE, IN CONTRAST, THAT IT WOULD TAKE THE ELECTRON (OR ANY OTHER MATERIAL OBJECT) $t = c/g \simeq 30\times10^6\,s$ ($1\,YEAR \simeq 30\times10^6$ SECONDS) TO REACH THE SPEED OF LIGHT UNDER FREE FALL WITH $g \cong 10\,m/s^2$.