Prepared by: PIEKAREWICZ
Date: MAR 12 2003

HOMEWORK 8 - PROBLEM 23

LET US DEFINE BOTH EVENTS FROM SOPHIE'S REFERENCE FRAME

X_1^{μ} = SOPHIE CRYING OUT IN PAIN = $(0,0)$
X_2^{μ} = TWIN BROTHER HITS THUMB = $(0,500)$

FOR THE SKEPTICAL SCIENTIST THESE EVENT OCCURRED AT $\tilde{X}^{\mu}=\Lambda^{\mu}_{\ \nu}X^{\nu}$

OR,

$$\begin{pmatrix} c\tilde{t}_1 \\ \tilde{X}_1 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct_1=0 \\ X_1=0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} ; \quad \beta=\frac{12}{13}, \gamma=\frac{13}{5}$$

AND

$$\begin{pmatrix} c\tilde{t}_2 \\ \tilde{X}_2 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct_2=0 \\ X_2=500 \end{pmatrix} = \begin{pmatrix} -\beta\gamma X_2 \\ \gamma X_2 \end{pmatrix}$$

THEN, ACCORDING TO THE SCIENTIST:

BROTHER HITS THUMB AT: $\tilde{t}_2 = -\beta\gamma\frac{X_2}{c} = -\frac{12}{5}\frac{X_2}{c} = -4\times10^{-3}\,s$

SOPHIE CRIES IN PAIN AT: $\tilde{t}_1 = 0$

THAT IS, BROTHER HITS HIS THUMB 4 ms BEFORE SOPHIE CRIES OUT.

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HOMEWORK 8 – PROBLEM 24

a)

$$\boxed{\begin{pmatrix}\tilde{t}\\ \tilde{x}\\ \tilde{y}\\ \tilde{z}\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\ -v & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}t\\ x\\ y\\ z\end{pmatrix}}$$

b) DEFINE (AS IN THE BOOK) $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$. THEN,

$$\boxed{\begin{pmatrix}\tilde{x}^0\\ \tilde{x}^1\\ \tilde{x}^2\\ \tilde{x}^3\end{pmatrix} = \begin{pmatrix}\gamma & 0 & -\beta\gamma & 0\\ 0 & 1 & 0 & 0\\ -\beta\gamma & 0 & \gamma & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}}$$

c) FOR THE COMBINED TRANSFORMATION:

$$\boxed{\begin{aligned}\begin{pmatrix}\tilde{x}^0\\ \tilde{x}^1\\ \tilde{x}^2\\ \tilde{x}^3\end{pmatrix} &= \begin{pmatrix}\bar{\gamma} & 0 & -\bar{\beta}\bar{\gamma} & 0\\ 0 & 1 & 0 & 0\\ -\bar{\beta}\bar{\gamma} & 0 & \bar{\gamma} & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}\\ &= \begin{pmatrix}\bar{\gamma}\gamma & -\bar{\gamma}\beta\gamma & -\bar{\beta}\bar{\gamma} & 0\\ -\beta\gamma & \gamma & 0 & 0\\ -\bar{\beta}\bar{\gamma}\gamma & \bar{\beta}\bar{\gamma}\beta\gamma & \bar{\gamma} & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}\end{aligned}}$$

THE ORDER OF THE TRANSFORMATION DOES MATTER; MATRICES, IN GENERAL, DO <u>NOT</u> COMMUTE.

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Homework 8 - Problem 25

(a)

$$X_A^{\mu} = (15, 5, 3, 0) \; ; \quad X_B^{\mu} = (5, 10, 8, 0)$$

The invariant interval is given by

i) $I \equiv \Delta X^{\mu} \Delta X_{\mu}$; $\Delta X^{\mu} \equiv X_A^{\mu} - X_B^{\mu} = (10, -5, -5, 0)$

or

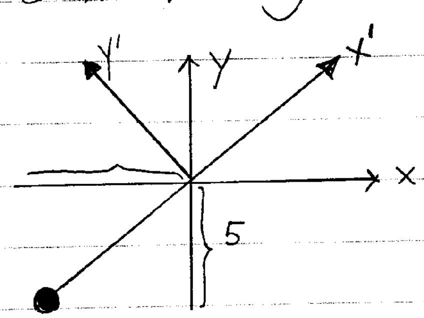
$$I = (\Delta x^0)^2 - \Delta\vec{x}\cdot\Delta\vec{x} = 100 - [25+25] = +50$$

or

$$\boxed{I = \Delta x^{\mu} \Delta x_{\mu} = 50}$$

ii) The event is timelike ($I > 0$; this is my convention) if such frame would exist (i.e., one with $\Delta\tilde{x}^0 = 0$) then the event would be spacelike ($I < 0$) in that frame; this is impossible as the interval is a Lorentz invariant.

iii) We must find a frame in which the four-vector Δx^{μ} becomes equal to $\Delta\tilde{x}^{\mu} = (\sqrt{50}, 0, 0, 0)$. Since distances perpendicular to the direction of motion do not change, the direction of the velocity must be in the x-y plane



In the "primed" coordinate system Δx^{μ} becomes equal to:

$$\Delta x^{\mu} \to \Delta x'^{\mu} = (10, -\sqrt{50}, 0, 0)$$
$$\Delta\tilde{x}^{\mu} \to \Delta\tilde{x}'^{\mu} = (\sqrt{50}, 0, 0, 0)$$

Prepared by:

Date:

IT IS NOW SIMPLE TO OBTAIN THE LORENTZ TRANSFORMATION:

$$\Delta \tilde{X}'^{\mu} :: \begin{pmatrix} \sqrt{50} \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 10 \\ -\sqrt{50} \end{pmatrix}$$

OR

$$\sqrt{50} = 10\gamma + \sqrt{50}\,\beta\gamma$$

$$0 = 10\beta\gamma + \sqrt{50}\,\gamma \Rightarrow \beta = -\frac{\sqrt{50}}{10} = -\frac{1}{\sqrt{2}}$$

AND

$$\gamma = (1-\beta^2)^{-1/2} = \left(1-\frac{1}{2}\right)^{-1/2} = \sqrt{2}$$

(NOTE THAT $10\gamma + \sqrt{50}\,\beta\gamma = 10\sqrt{2} - \sqrt{50} = 2\sqrt{50} - \sqrt{50} = \sqrt{50}$ AS IT SHOULD BE).

IN THIS WAY THE REQUIRED VELOCITY IS GIVEN BY

$$\boxed{\vec{\beta} = -\frac{1}{\sqrt{2}}\hat{x}' = -\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{y}\right) = -\frac{1}{2}(\hat{x}+\hat{y})}$$

b)

$$X_A^{\mu} = (1,2,0,0) \quad \text{AND} \quad X_B^{\mu} = (3,5,0,0)$$

i) $\Delta X'^{\mu} = X_B^{\mu} - X_A^{\mu} = (2,3,0,0) \rightarrow I = \Delta X^{\mu}\Delta X_{\mu} = 4-9 = -5$

OR

$$\boxed{I = \Delta X^{\mu}\Delta X_{\mu} = -5}$$

ii) THE INTERVAL IS SPACELIKE SO WE SHOULD BE ABLE TO FIND A FRAME IN WHICH BOTH EVENTS OCCUR SIMULTANEOUSLY; NAMELY A FRAME FOR WHICH $\Delta \tilde{X}^{\mu} = (0, \sqrt{5})$.

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THUS,

$$\Delta \tilde{X}'' = \begin{pmatrix} 0 \\ \sqrt{5} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

OR

$$0 = 2\gamma - 3\beta\gamma \Rightarrow \beta = \frac{2}{3} \text{ AND } \gamma = (1-\beta^2)^{-1/2} = \frac{3}{\sqrt{5}}$$

$$\sqrt{5} = -2\beta\gamma + 3\gamma$$

(NOTE THAT $-2\beta\gamma + 3\gamma = -2 \cdot \frac{2}{\sqrt{5}} + \frac{9}{\sqrt{5}} = \sqrt{5}$, AS REQUIRED).

THE REQUIRED VELOCITY IS GIVEN BY $\boxed{\vec{\beta} = \frac{2}{3}\hat{x}}$

iii) SINCE THE EVENT IS SPACELIKE, THERE IS NO FRAME IN WHICH THEY OCCUR AT THE SAME POINT.

HOMEWORK 9 – PROBLEM 26

IN THE REST-FRAME OF THE ELECTRON ITS FOUR-MOMENTUM IS GIVEN BY,

$$p_0^{\mu} = \begin{pmatrix} E = m_0c^2 \\ \vec{0} \end{pmatrix}$$

TO OBTAIN IT'S FOUR-MOMENTUM IN THE LABORATORY FRAME WE SIMPLY APPLY A LORENTZ TRANSFORMATION WITH $\vec{\beta} = \frac{\vec{V}}{c} = \frac{V}{c}\hat{x}$. THAT IS,

$$\begin{pmatrix} E \\ pc \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} m_0c^2 \\ 0 \end{pmatrix}$$

OR

$$E = m_0\gamma c^2 \quad \text{AND} \quad pc = m_0\gamma\beta c^2 = m_0\gamma Vc$$

OR

$$\boxed{E = m_0\gamma c^2 \quad \text{AND} \quad p = m_0\gamma V}$$

NOW NOTE THAT THE QUANTITY $p \cdot p = p^{\mu} g_{\mu\nu} p^{\nu} = p^{\mu} p_{\mu}$ IS A RELATIVISTIC OR LORENTZ INVARIANT; THAT IS,

$$\begin{aligned} p \cdot p = E^2 - (pc)^2 &= (m_0\gamma c^2)^2 - (m_0\gamma Vc)^2 \\ &= m_0^2c^4\gamma^2(1-\beta^2) = m_0^2c^4 \end{aligned}$$

OR

$$\boxed{p \cdot p = E^2 - (pc)^2 = (m_0c^2)^2 \text{ IS FRAME INDEPENDENT}}$$

PREPARED BY PIEKAREWICZ
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HOMEWORK 9 - PROBLEM 27

LABORATORY FRAME

TO COMPUTE THE SPEED OF ELECTRON 1 IN THE REST FRAME OF ELECTRON 2 WE MUST "BOOST" THE LABORATORY FRAME BY $+\beta_2$. IN THIS CASE THE LORENTZ TRANSFORMATION BECOMES:

$$A(\beta_2) = \begin{pmatrix} \gamma_2 & \beta_2\gamma_2 \\ \beta_2\gamma_2 & \gamma_2 \end{pmatrix}$$

CLEARLY, UNDER THIS LORENTZ TRANSFORMATION THE FOUR-MOMENTUM OF THE ELECTRON 2 BECOMES EQUAL TO: $p_2'^{\mu} = \begin{pmatrix} m_0 c^2 \\ 0 \end{pmatrix}$ WHILE THAT OF

ELECTRON 1 BECOMES,

$$p_1'^{\mu} = \begin{pmatrix} E_1' \\ p_1' c \end{pmatrix} = \begin{pmatrix} \gamma_2 & \beta_2\gamma_2 \\ \beta_2\gamma_2 & \gamma_2 \end{pmatrix} \begin{pmatrix} m_0\gamma_1 c^2 \\ m_0\gamma_1\beta_1 c^2 \end{pmatrix}$$

$$= \begin{pmatrix} m_0\gamma_1\gamma_2 c^2 (1+\beta_1\beta_2) \\ m_0\gamma_1\gamma_2 c^2 (\beta_2+\beta_1) \end{pmatrix}$$

THUS,

$$\boxed{\beta_1' = \frac{p_1' c}{E_1'} = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}}$$

THIS IS NOTHING BUT THE LAW OF ADDITION OF VELOCITIES.

PRETTY EASY COMPS. PROBLEM!

PREPARED BY PIEKAREWICZ
DATE MAR 2 7 2002

HOMEWORK 9 – PROBLEM 28

THE LIFETIME OF THE PION AS SEEN BY A TERRESTRIAL OBSERVER IS TIME DILATED BY A FACTOR OF

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\left(\frac{99}{100}\right)^2}} = 7.09$$

THUS, AS SEEN BY A TERRESTRIAL OBSERVER THE PION TRAVELS A DISTANCE OF:

$$\boxed{d = (0.99c) \times (7.09\tau) = 54.75m}$$