Qualifying Exam- Fall 2006
Electrodynamics
Problem 1

Twelve wires, each of resistance r, are connected to form the edges of a cube. Calculate the effective resistance R of this network across a body-diagonal of the cube.

Qualifying Exam 2006
Quantum Mechanics
Problem 2

Consider the three spin-1 matrices

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \qquad S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}; \qquad S_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Calculate the commutator of S_x and S_y.

(b) What are the possible values we can get if we measure the spin along the x-axis?

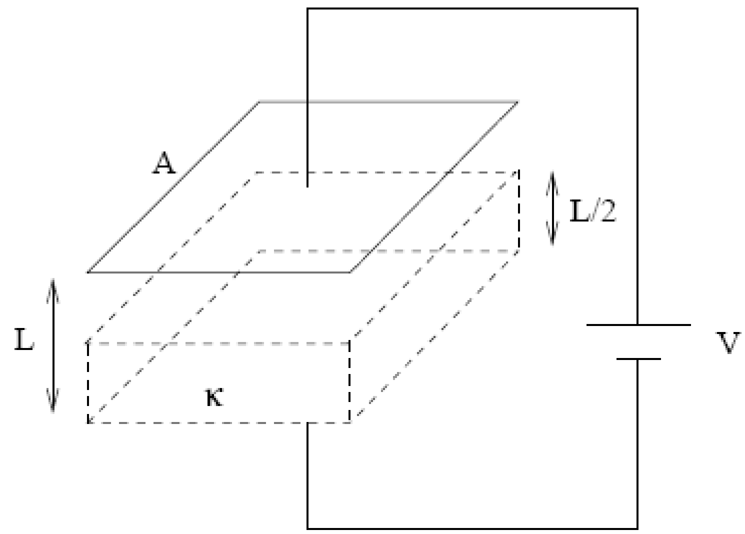
(c) Suppose we obtain the largest possible value when we measure the spin along the x-axis. If we now measure the spin along the z-axis, what are the probabilities for the various outcomes?

Qualifying Exam- Fall 2006
Classical Mechanics (Escape Velocity)
Problem 3

Suppose the Moon were to have the same mass as the Earth, and you are trying to throw one of your physics books from the Earth to the Moon. With what minimum velocity must the book leave the surface of the Earth? Neglect the relative motion of the Earth and them Moon, and the Earth's rotation. The mass of the Earth is ME = 6.0·1024 kg, the radius of the Earth is 6.4 · 106 m, and the distance from the center of the Earth to the center of the Moon is REM = 3.8 · 108 m. Compare your answer to the escape velocity from Earth alone. The gravitational constant is G = 6.67 · 10−11 N m2/kg2.

Qualifying Exam-Fall 2006
Electrodynamics
Problem 4

Consider a capacitor connected to a battery of voltage V. Let the capacitor have an area A, and distance L between the plates. Assume that the capacitor has a layer of dielectric (of dielectric constant κ, so that $\varepsilon = \kappa\varepsilon_o$) of thickness L/2 on the lower plate, as shown in the figure.



a) Calculate the capacitance of the capacitor.

b) Calculate the charge on the capacitor.

c) Calculate the value of the electric displacement D in the capacitor.

d) Calculate the value of the electric field inside the dielectric layer, and in the air above it.

e) Calculate the electrostatic energy stored in the system. How would it change if the dielectric is removed?

Qualifying Exam Fall-2006
Modern Physics
Problem 5

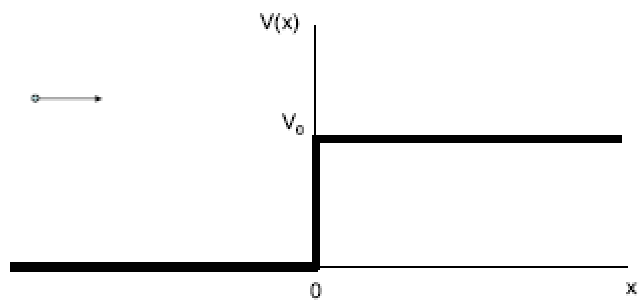
A monochromatic particle beam consist of particles whose total energy is 100 times their rest mass. The rest lifetime of the particles is 0.10 ns. In the laboratory, the distance between the point where the particles are generated and the detector is 6.0 m. What fraction of the generated particles reach the detector.

Qualifying Exam-Fall 2006
Quantum Mechanics
Problem 6

Consider a one-dimensional step potential of the form:

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x \geq 0 \end{cases}$$

where $V_0 > 0$. A quantum particle with mass m and energy $E > V_0$ is incident on this step "from the left" as shown in the figure.



(a) Write down the appropriate solutions of the time-independent Schrödinger equation for this particle in the x < 0 region and the x > 0 region.
(b) Apply the appropriate boundary conditions at the point x = 0 to match these solutions.
(c) Derive expressions for the probabilities that the particle is reflected (R) and transmitted (T) by the step.
Hint: Recall that the probability density current is given by:

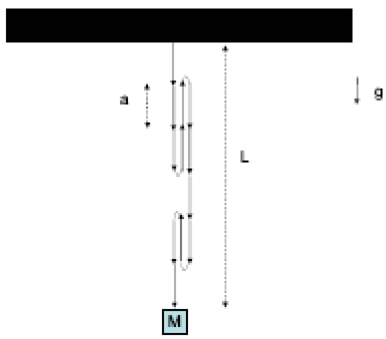
$$j(x) = \frac{\hbar}{2im}\left(\Psi^* \frac{\partial \Psi}{\partial x} - \Psi \frac{\partial \Psi^*}{\partial x}\right).$$

Qualifying Exam Fall-2006
Thermodynamic/Statistical Mechanics
Problem 7

A simple statistical mechanics model of a rubber band consists of N connected rigid segments, each of length a, which can either point up or down. The total length L of the rubber band is then:

L=a[(# of segments pointing down)-(# of segments pointing up)].

One end of this rubber band is fixed, and a mass M is attached to the other end so that the rubber band hangs vertically (see figure).



(a) Find an expression for the entropy S of this rubber band as a function of its length L. (Hint: Determine the number of microstates Ω, equal to the number of distinct ways the rigid segments can point up or down, when the rubber band has length L, and use the fact that $S = k_B \ln \Omega$, where k_B is Boltzmann's constant).

(b) Find an expression for the free energy of this rubber band as a function of L and the temperature T. Recall that the free energy is F(L,T)= E(L)-TS(L), where, in this case, E(L) is the gravitational potential energy of the mass M (you may ignore the mass of the rubber band) and S(L) is the entropy you found in part (a).

(c) Minimize the free energy with respect to L to obtain the equilibrium length of the rubber band as a function of T, M, g, a and k_B. (Hint: Work in the limit of large N and use Stirling's approximation: $\ln N! \cong N \ln N - N$.)

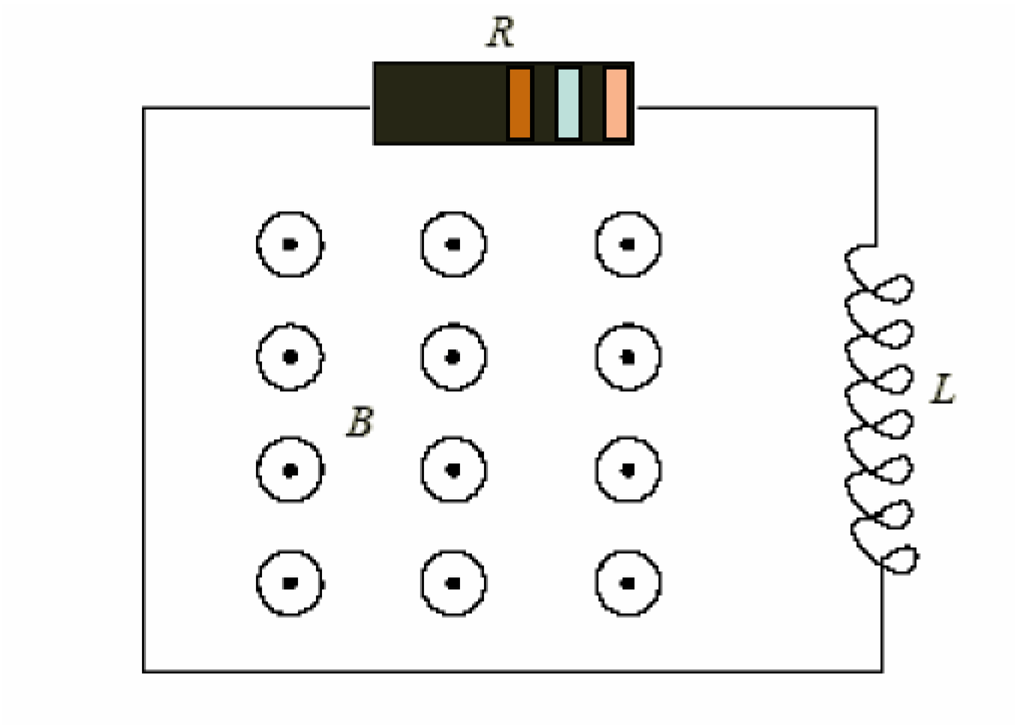
(d) If the temperature is increased, does the rubber band get longer or shorter?

Qualifying Exam Fall-2006
Electrodynamics
Problem 8

A loop of wire of resistance R and a coil of self-inductance L encloses an area A. A spatially uniform magnetic field is applied perpendicular to the plane of the loop with the following time dependence: t < 0 the field is zero; for 0 < t < t_0, B(t) = kt, while for t > t_0, the field now remains constant at $B_0 = kt_0$.

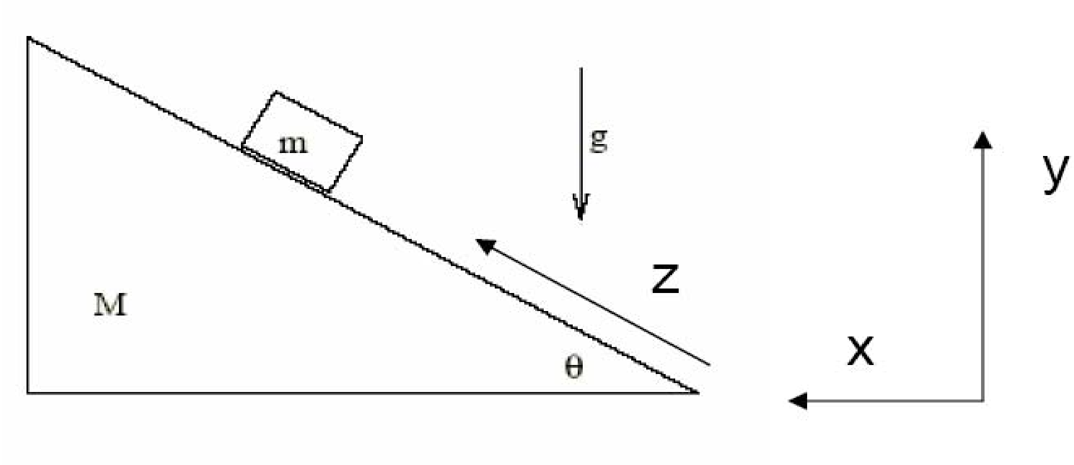
a) Calculate the current I in the loop for all times t > 0, given that I = 0 for t = 0.

b) Make simple sketches of the current vs. time for t_0 < R/L and t_0 >> R/L.



Qualifying Exam Fall-2006
Classical Mechanics
Problem 9

A wedge of mass M=4.5 kg sits on a horizontal surface. Another mass m=2.3 kg sits on the sloping side of the wedge. The incline is at an angle θ=31.7 degrees with respect to the horizontal, and g=9.8 m/s^2. All surfaces are frictionless. The mass m is released from rest on mass M, which is also initially at rest. Use Lagrangian mechanics and the suggested coordinates to determine the vector values of the accelerations of both M and m once the mass is released.



Qualifying Exam Fall-2006
Thermodynamics/ Statistical Mechanics
Problem 10

A system of N bosons in two dimensions has an energy-momentum relationship:

$$E = cp^{3/2}$$

and density n=N/A (A is the area).

(a) What is Bose-Einstein condensation?
(b) Show that at low temperatures, the system will Bose condense, and that the Bose condensation temperature $T_C \sim n^{\alpha}$. Find α.
(c) Show that the entropy below T_C goes like $S \sim T^{\beta}$, and the "pressure" (equivalent in two dimensions) goes as $P \sim T^{\gamma}$. Find β and γ.
Hint: the differential of the grand potential is:

$$d\theta = -SdT - PdA + nd\mu .$$

Qualifying Exam Fall-2006
Modern Physics
Problem 11

Describe briefly, with equations or sketches as needed, the following famous effects or experiments:

(a) The Auger Effect
(b) Bragg diffraction
(c) Rutherford scattering
(d) The Mössbauer effect
(e) The Stern-Gerlach experiment

Qualifying Exam Fall-2006
Quantum Mechanics
Problem 12

The time-independent wave function for a particle of mass *m* which moves in a one-dimensional potential *V(x)* has the form $\psi(x) = A\exp\left|-a^2x^2\right|$, where $a = \sqrt{m\omega/2\hbar}$ and A is normalization constant.

a. Using the time-independent Schrodinger equation find *V(x)* and the energy eigenvalue for $\psi(x)$.

b. Identify the system. Which one of its quantum states is described by $\psi(x)$?