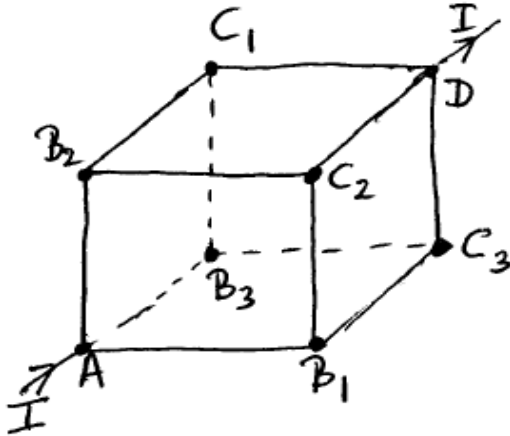


Qualifying Exam Fall 2006 Solutions

Solution 1



-The AD-axis of the cube has threefold symmetry, i.e. the cube is invariant under rotations by $\pm 120^\circ$ about that axis.

-Hence, the corners B_1, B_2 and B_3 are equivalent and have the same potential.

-Also, the corners C_1, C_2 and C_3 are equivalent and have the same potential.

If a current I enters A , then:

- A current $\frac{I}{3}$ circulates through each of the resistors AB_i .
- A current $\frac{I}{6}$ circulates through each of the six resistors B_iC_j .
- A current $\frac{I}{3}$ passes through each of the three resistors C_jD .
- The potential drop between A and B_i is $\frac{rI}{3}$.
- The potential drop between B_i and C_j is $\frac{rI}{6}$.
- The potential drop between C_j and D is $\frac{rI}{6}$.

The potential difference between A and D is:

$$rI \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{6} \right) = \frac{5}{6} rI$$

Hence $R = \frac{5}{6} r$

Solution 2

a.)

$$\begin{aligned}
 [S_x, S_y] &= \frac{\hbar^2}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} - \frac{\hbar^2}{2} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
 &= \frac{\hbar^2}{2} \begin{pmatrix} i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & -i \end{pmatrix} - \frac{\hbar^2}{2} \begin{pmatrix} -i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & i \end{pmatrix} = i\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = i\hbar S_z
 \end{aligned}$$

b.)

A measurement along the x-axis has as an outcome, one of the eigenvalues of S_x .

$$\det \begin{pmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\lambda & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\lambda \end{pmatrix} = 0 \qquad \begin{aligned} -\lambda^3 + 2\lambda \frac{\hbar^2}{2} &= 0 \\ \lambda &= 0, \pm\hbar \end{aligned}$$

The possible values are: $\hbar, 0, -\hbar$.

c.)

The largest possible value along the x-axis is \hbar ; obtain eigenvector in the basis of quantization along the z-axis:

$$\begin{pmatrix} -\hbar & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\hbar & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\hbar \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \qquad a = \frac{b}{\sqrt{2}} = c$$

The normalized eigenvector: $\frac{1}{2}(|+1\rangle + \sqrt{2}|0\rangle + |-1\rangle)$

Probabilities: $P(+1) = \frac{1}{4}$ $P(0) = \frac{1}{2}$ $P(-1) = \frac{1}{4}$

Solution 3

Conservation of energy given by the sum of potential energy due to gravity and kinetic energy can be used to determine escape velocity. In the case of Earth along the potential is given by:

$$\phi(r) = -G \frac{M_E m}{r}$$

where m is the mass of the book. The book will escape if initial kinetic energy is high enough to overcome the potential at $r = R_E$. Thus

$$\frac{mv_E^2}{2} = G \frac{M_E m}{R_E} \text{ thus } v_E = \sqrt{\frac{2GM_E}{R_E}} = 11 \text{ km/s}$$

In the Earth-Moon case the potential is

$$\phi(r) = -G \frac{M_E m}{r} - G \frac{M_M m}{|R_{EM} - r|}$$

where $M_M = M_E$. The potential is a symmetric double-well and in order to leave the surface of the earth the kinetic energy must be high enough to overcome a saddle point right in the middle between earth and moon. Thus the condition for escape velocity is

$$\frac{mv_E^2}{2} - GM_E m \left(\frac{1}{R_E} + \frac{1}{R_{EM} - R_E} \right) = -\frac{4GM_E m}{R_{ME}}$$

This equation solved for escape velocity gives $v_E = 7.7 \text{ km/s}$.

Solution 4

a) Calculate the capacitance of the capacitor.

To find the capacitance we would like to find the voltage between the plates for a given charge $+Q$ and $-Q$ on the two plates. We can use Gauss's law (both with and without a dielectric) to find the electric field due to the charge on the plates, using the approximation that there are no end effects. Gauss's law reads, in general

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{enclosed}}{\epsilon}$$

where $\epsilon = \kappa\epsilon_0$ is the general form of the electric permittivity in the presence of a dielectric (note that in free space $\kappa = 1$).

If we use a pill-box shaped Gaussian surface with end area A' and with one end below the bottom plate, which is charged to $-Q$, and the other end inside the dielectric, then Gauss's law gives us

$$\oint \vec{E}_d \cdot d\vec{S} = E_d A' = \frac{Q_{enc}}{\epsilon} = \frac{\sigma A'}{\epsilon} = \frac{Q}{\epsilon A},$$

where E_d is the field inside the dielectric, since the field is zero outside the electric on the bottom end of the pill-box and always perpendicular to the sides of the pill-box. This gives

$$E_d = \frac{Q}{\epsilon A} = \frac{Q}{\kappa\epsilon_0 A}.$$

Exactly the same procedure, but with the two ends of the pillbox above and below the top plate in empty space, results in the field between the dielectric and the top plate of

$$E = \frac{Q}{\epsilon_0 A} = \frac{Q}{\epsilon_0 A}.$$

We can now find the relation between the voltage difference V between the bottom and top plates and Q by integrating the constant electric field (which points straight up) along a vertical line from the bottom plate to the top plate

$$V = \int_0^{L/2} \vec{E}_d \cdot d\vec{l} + \int_{L/2}^L \vec{E} \cdot d\vec{l} = \frac{Q}{\kappa\epsilon_0 A} \frac{L}{2} + \frac{Q}{\epsilon_0 A} \frac{L}{2} =$$

so that we have

$$C = \frac{Q}{V} = \frac{2\epsilon_0 A}{L} \left(\frac{1}{\kappa} + 1 \right)^{-1} = \frac{2\epsilon_0 A}{L} \left(\frac{\kappa}{1 + \kappa} \right).$$

Note that this is just the capacitance $\frac{2\epsilon_0 A}{L}$ of a free-space parallel-plate capacitor with area A and plate gap $L/2$ in series with that of the same capacitor with dielectric inserted, which is larger, $2\kappa\epsilon_0 AL$. The total capacitance of two capacitors in series is like that of resistors in parallel, i.e. it is given by

$$C = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}.$$

b) Calculate the charge on the capacitor.

For any capacitor $Q = CV$, so

$$Q = \frac{2\epsilon_0 AV}{L} \left(\frac{\kappa}{1 + \kappa} \right).$$

c) Calculate the value of the electric displacement D in the capacitor.

The source of the electric displacement D is free charges, so we are to ignore any surface charges on the dielectric. This means that the electric displacement everywhere in the capacitor is just the electric field in the free space region,

$$D = E = \frac{Q}{\epsilon_0 A} = \frac{2V}{L} \left(\frac{\kappa}{1 + \kappa} \right)$$

and it points from the lower plate to the upper plate.

d) Calculate the value of the electric field inside the dielectric layer, and in the air above it. With our approach we have already done this and we have

$$E_d = \frac{Q}{\kappa\epsilon_0 A} = \frac{2V}{L(1 + \kappa)}, \quad E = \frac{Q}{\epsilon_0 A} = D.$$

e) Calculate the electrostatic energy stored in the system. How would it change if the dielectric is removed? The electrostatic energy stored in a capacitor is just

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2$$

and so we see that

$$U = \frac{1}{2} \frac{2\varepsilon_o A}{L} \left(\frac{\kappa}{1+\kappa} \right) V^2 = \frac{\varepsilon_o AV^2}{L} \left(\frac{\kappa}{1+\kappa} \right).$$

Obviously if κ is reduced to unity by removing the dielectric but the applied voltage remains the same, the capacitance is reduced and so the amount of stored energy is reduced.

Solution 5

To find the fraction of particles which reach the detector, we have to find the time that it takes for particles to reach the detector. Let N_0 be the number of the generated particles and N be the number of particles which reach the detector. The relation between them is given by:

$$N = N_0 \exp\left(-\frac{t}{\tau}\right). \quad (1)$$

Eq.1 is the formula which is used to find the number of particles after time t when their life time is τ . Therefore we have to find t and τ .

The generated particles have a life time $\tau_0 = 100$ ns, in their rest frame. This means that if we “travel” with the particle the particle will decay after 10ns. But this is not the time which is measured in the laboratory. Since the rest frames of the particles move with respect to the lab-frame. According to the special relativity there is dilation of time between the time measured in the lab-frame and the one in the rest frame of the particle. The measured time τ is related to τ_0 by:

$$\tau = \gamma \tau_0 = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (2)$$

where v is the velocity of the particles. Here the velocity of the particles is unknown. To find the velocity we use the information given in the problem. We know the total energy of the particle. The relation between the total energy and the velocity is given by:

$$E^2 = p^2 c^2 + m_0^2 c^4 \quad (3)$$

but, $p = \gamma m_0 v$

$$\begin{aligned} E^2 &= \gamma^2 m_0^2 c^2 + m_0^2 c^4 \\ &= [\gamma^2 v^2 + c^2] m_0^2 c^2 \end{aligned}$$

using the definition of $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$= \gamma^2 m_0^2 c^4$$

$$\text{or } E = \gamma m_0 c^2 \quad (5)$$

The total energy is:

$$E = 100m_0c^2 \Rightarrow \gamma = 100 \quad (6)$$

Substitute Eq.6 in Eq.2 we obtain:

$$\tau = 10^{-8} \text{ s}. \quad (7)$$

From Eq.6 and the definition of γ we get:

$$v \approx c \quad (8)$$

The time for particles to reach the detector is given by:

$$t = \frac{1}{v} \approx \frac{1}{c} = 2 \times 10^{-8} \text{ s}. \quad (9)$$

Substitute Eq.7 and 9 in Eq.1 it yields:

$$\frac{N}{N_0} = e^{-2} \approx 13.5\%. \quad (10)$$

Solution 6

(a) For $x < 0$ the time-independent Schrödinger equation is:

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x) = E\Psi(x),$$

for which the solution including both the transmitted and reflected waves is:

$$\Psi_{x<0}(x) = Ae^{ikx} + Be^{-ikx}; \quad k = \sqrt{2mE}/\hbar.$$

For $x > 0$ the Schrödinger equation is:

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x) + V_0\Psi(x) = E\Psi(x),$$

and the solution is:

$$\Psi_{x>0}(x) = Ce^{ik'x}; \quad k' = \sqrt{2m(E - V_0)}/\hbar,$$

where we keep only the right moving (transmitted) wave.

(b) Applying the boundary conditions at $x = 0$ that both Ψ and its first derivative with respect to x must be continuous yields:

$$\Psi_{x<0}(0) = \Psi_{x>0}(0) \Rightarrow A + B = C,$$

and

$$\Psi'_{x<0}(0) = \Psi'_{x>0}(0) \Rightarrow ik(A - B) = ik'C,$$

from which we find

$$C = \frac{2k}{k + k'} A, \quad B = \frac{k - k'}{k + k'} A.$$

(c) The probability currents for the incident (j_i), reflected (j_r) and transmitted (j_t) waves are easily found to be:

$$j_i = \frac{\hbar k}{m} |A|^2, \quad j_r = -\frac{\hbar k}{m} |B|^2, \quad j_t = \frac{\hbar k'}{m} |C|^2.$$

The reflection coefficient is then:

$$R = \frac{j_r}{j_i} = \frac{|B|^2}{|A|^2} = \frac{(k - k')^2}{(k + k')^2},$$

and the transmission coefficient is

$$T = \frac{j_t}{j_i} = \frac{k'}{k} \frac{|C|^2}{|A|^2} = \frac{4kk'}{(k + k')^2}.$$

(As a check we see that $T+R=1$, as it must.)

Solution 7

(a) If n is the number of segments which point up and L is the length of the rubber band, then:

$$\frac{L}{a} = N - 2n \Rightarrow n = \frac{N}{2} - \frac{L}{2a}$$

The number of microstates corresponding to the length L is then N choose n , i.e.

$$\# \text{ of microstates} = \binom{N}{n} = \frac{N!}{(N-n)!n!} = \frac{N!}{\left(\frac{N}{2} + \frac{L}{2a}\right)! \left(\frac{N}{2} - \frac{L}{2a}\right)!}$$

From this, we find the entropy is:

$$S = k_B \ln \frac{N!}{\left(\frac{N}{2} + \frac{L}{2a}\right)! \left(\frac{N}{2} - \frac{L}{2a}\right)!}$$

(b) The energy of the system is simply $E(L) = -mgL$. Thus, the free energy at temperature T is:

$$F = -mgL - k_B T \ln \frac{N!}{\left(\frac{N}{2} + \frac{L}{2a}\right)! \left(\frac{N}{2} - \frac{L}{2a}\right)!}$$

(c) For $N \gg 1$, Stirling's approximation tells us that:

$$\frac{S}{k_B} \cong N \ln N - \left(\frac{N}{2} + \frac{L}{2a}\right) \ln \left(\frac{N}{2} + \frac{L}{2a}\right) - \left(\frac{N}{2} - \frac{L}{2a}\right) \ln \left(\frac{N}{2} - \frac{L}{2a}\right)$$

Thus

$$\frac{\partial S}{\partial L} = \frac{k_B}{2a} \ln \frac{Na - L}{Na + L}$$

So, minimizing the Free energy we have

$$\frac{\partial F}{\partial L} = -mg - k_B T \frac{1}{2a} \ln \frac{Na - L}{Na + L} = 0$$

and, solving for L yields

$$L = Na \tanh \frac{mga}{k_B T}$$

(d) As T increases the rubber band gets shorter. This is seen directly from the solution of Part (c), but also from the fact that the higher temperatures favor higher entropy configurations and hence smaller rubber band lengths.

Solution 8

The flux Φ is given by

$$\Phi = BA = \begin{cases} ktA & t < t_0 \\ kt_0A & t > t_0 \end{cases}$$

Therefore we have

$$\mathcal{E} = -\frac{d\Phi}{dt} = \begin{cases} kA & t < t_0 \\ 0 & t > t_0 \end{cases}$$

where the minus sign reminds us that the current flows to oppose the increasing B, or in the clockwise direction. The total voltage Kirchoff equation is

$$V_R + V_L = |\mathcal{E}|$$

$$IR + L \frac{dI}{dt} = |\mathcal{E}|$$

$$\text{Or } \frac{dI}{dt} = |\mathcal{E}| - \frac{IR}{L}$$

The general solution of this differential equation may easily be obtained by the substitution $u = |\mathcal{E}| - \frac{IR}{L}$, which yields $u = u_0 \exp(-\frac{Rt}{L})$ where $u_0 = |\mathcal{E}|$, and hence

$$I = \frac{|\mathcal{E}|}{R} (1 - u_0 \exp(-\frac{Rt}{L})).$$

Or, following the conventions of the problems class:

We can solve this by adding a particular solution $I_p = \frac{|\mathcal{E}|}{R}$ to the solution

$I_h = k \exp\left(-\frac{Rt}{L}\right)$ of the homogenous equation $L \frac{dI}{dt} = -RI$, and then matching to initial conditions which are $I(t=0) = 0$, we have:

$$I = \frac{|\mathcal{E}|}{R} + k \exp\left(-\frac{Rt}{L}\right)$$

$$I(0) = \frac{|\mathcal{E}|}{R} + k \Rightarrow k = -\frac{|\mathcal{E}|}{R}$$

$$I(t) = \frac{kA}{R} \left(1 - \exp\left(-\frac{Rt}{L}\right)\right), \quad 0 < t < t_0$$

For $t > t_0$ there is only an exponential decay of the current $I(t_0) = \frac{KA}{R} \left(1 - \exp\left(-\frac{Rt_0}{L}\right)\right)$.

This time the differential equation is:

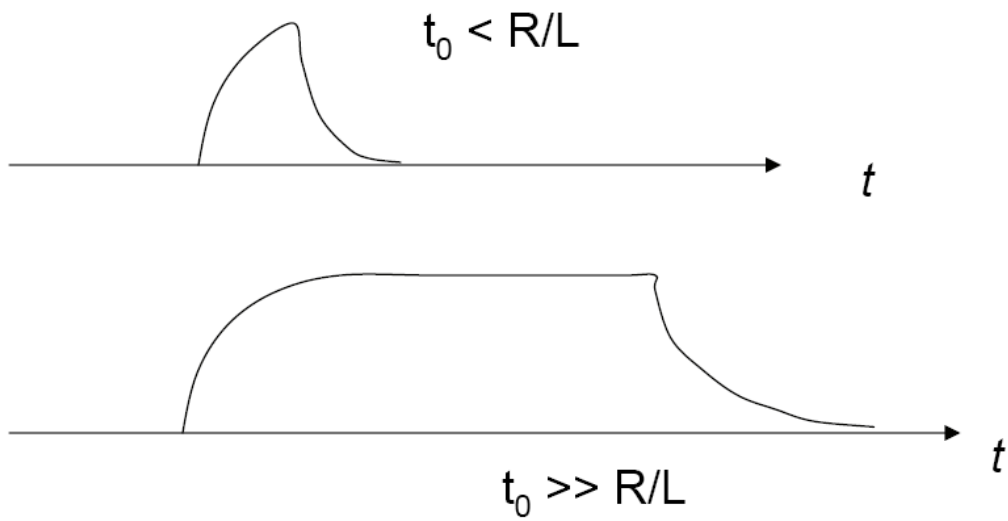
$$\frac{dI}{dt} = -\frac{IR}{L}$$

With the immediate solution $I = I_0 \exp\left(-\frac{(t-t_0)R}{L}\right)$ since now the limits of the integration are from $I(t_0)$ to $I(t)$ and t_0 to t .

Hence we have.

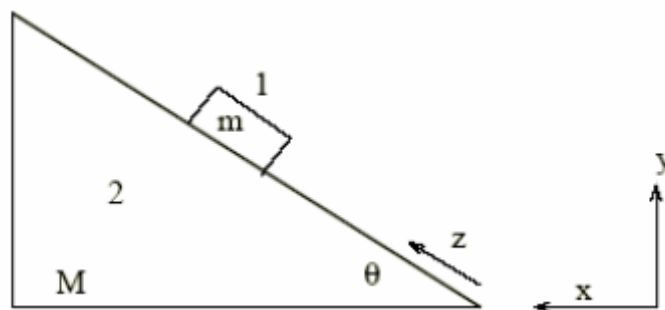
$$I(t) = \frac{KA}{R} \left(1 - \exp\left(-\frac{Rt_0}{L}\right)\right) \exp\left(-\frac{R(t-t_0)}{L}\right)$$

$$I(t) = \frac{KA}{R} \left(\exp\left(\frac{Rt_0}{L}\right) - 1\right) \exp\left(-\frac{Rt}{L}\right)$$



Solution 9

Introduce the generalized coordinates as in the figure below.



The Lagrangian for this system will be given by

$$L = T_{\text{wedge}} + T_m - V_{\text{wedge}} - V_m \quad (1)$$

and

$$T_{\text{wedge}} = \frac{1}{2} M \dot{x}_2^2 \quad (2)$$

$$V_{\text{wedge}} = Mgy_2 = \text{constant, since the wedge does not move in the } y \text{ direction.} \quad (3)$$

$$T_m = \frac{1}{2} m(\dot{x}_1^2 + \dot{y}_1^2) \quad (4)$$

$$V_m = mgy_1 \quad (5)$$

But x_1 and y_1 can be written as follows:

$$x_1 = x_2 + z_1 \cos(\theta) \quad (6)$$

$$y_1 = z_1 \sin(\theta) \quad (7)$$

Substitute Eqs.6 and 7 in Eqs.4 and 5, then replace T's and V's in Eq.1, and rename $x_2 = x$, $z_1 = z$ for simplicity. The result is given by:

$$L = \frac{1}{2} m[\dot{x}^2 + \dot{z}^2 + 2\dot{x}\dot{z} \cos(\theta)] + \frac{1}{2} M \dot{x}^2 - mg \sin(\theta) \quad (8)$$

The equations of motion are obtained by:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0, \text{ where } q = x, z. \quad (9)$$

We get:

$$m\ddot{x} + m\ddot{z} \cos(\theta) + M\ddot{x} = 0 \quad (10)$$

$$m\ddot{z} + m\ddot{x} \cos(\theta) + mg \sin(\theta) = 0 \quad (11)$$

From Eq.11 solve for \ddot{z} , and then substitute in Eq.10

$$\ddot{x} = \frac{mg \sin(\theta) \cos(\theta)}{m + M - \cos^2(\theta)}. \quad (12)$$

Substituting the numerical values for different parameters, we get:

$$\ddot{x} = 1.9624 \frac{m}{s^2} \quad (13)$$

$$\ddot{z} = -6.82 \frac{m}{s^2} \quad (14)$$

To find the acceleration of the mass m , derivate Eqs.6 and 7 twice and use the numerical values from Eq13 and 14

$$\ddot{x}_1 = -3.85 \frac{m}{s^2} \quad (15)$$

$$\ddot{y}_1 = -3.58 \frac{m}{s^2} \quad (16)$$

Solution 10

- (a) The abrupt accumulation of bosons in the ground state at temperatures below T_C is called Bose-Einstein condensation. This is a property of bosons which have no restriction to the number of particles per energy state (as opposed to fermions which must obey the Exclusion Principle.)
- (b) This can be solved in a few different ways. Here is the solution that the students have access to (which is correct):

For Bosons:

$$N = \sum_p \frac{1}{e^{\beta(E-\mu)} - 1} = \frac{A}{(2\pi)^2} \int d^2 p \frac{1}{e^{\beta(E-\mu)} - 1}$$

Bose-Einstein condensation occurs if N is finite for $\mu \rightarrow 0$. At small energies (and therefore small momenta) the integral becomes:

$$\int \frac{d^2 p}{e^{\beta E} - 1} = \int \frac{p dp}{1 + \beta E - 1} = \int \frac{p dp}{\beta c p^{3/2}} \sim \int \frac{dp}{p^{1/2}}$$

Since this integral is finite (over all momenta), we have Bose condensation.

To calculate T_C , we have to do the integral:

$$N = \frac{A}{(2\pi)^2} \int d^2 p \frac{1}{e^{\beta c p^{3/2}} - 1}$$

Change of variable: let $x = \beta c p^{3/2}$

Therefore: $p = (kT_C / c)^{2/3} x^{2/3}$

Our integral is now:

$$N = \frac{A}{(2\pi)^2} \left(\frac{kT_C}{c} \right)^{4/3} \left(\frac{2}{3} \right) \int dx \frac{x^{1/3}}{e^x - 1}$$

The integral just gives a number, so we have $N / A \sim (T_C)^{4/3}$ or $T_C \sim n^{3/4}$.

The exponent $\alpha=3/4$.

- (c) The differential of the grand potential is:

$$D\theta = -SdT - PdA + nd\mu$$

From this, we can calculate the entropy and pressure:

$$S = - \left(\frac{d\theta}{dT} \right)_{A,\mu}$$

$$P = -\left(\frac{d\theta}{dA}\right)_{T,\mu}$$

We can calculate the grand potential from:

$$\theta = -kT \ln Z$$

And the grand partition function:

$$Z = \sum e^{-\beta(E-\mu N)}$$

The grand partition function is a sum over particles and momenta:

$$Z = \prod_p \sum_n e^{-\beta(E-\mu)n} = \prod_p \frac{1}{1 - e^{-\beta(E-\mu)}}$$

Calculating the grand potential:

$$\theta = kT \sum_p \ln(1 - e^{-\beta(E-\mu)}) = \frac{kTA}{(2\pi)^2} \int d^2 p \ln(1 - e^{-\beta(E-\mu)})$$

Below T_C , $\mu=0$. We can evaluate the integral through a change of variables:

$$p = \left(\frac{kT}{c}\right)^{2/3} x^{2/3}$$

Therefore:

$$\theta = kT(T)^{4/3} A(\text{int}) \sim T^{7/3} A$$

Where, again, the integral just gives a number. We can now calculate:

$$S = -\frac{d\theta}{dT} \sim T^{4/3} \rightarrow \beta = 4/3$$

$$P = -\frac{d\theta}{dA} \sim T^{7/3} \rightarrow \gamma = 7/3$$

Solution 11

- (a) The Auger Effect: An atom with a missing inner electron can lose excitation energy by the Auger effect without emitting an x-ray photon. In the Auger effect an outer-shell electron is ejected from the atom at the same time that another outer-shell electron drops to the incomplete inner shell. Thus the ejected electron carries off the atom's excitation energy instead of a photon doing this. In a sense, the Auger effect represents an internal photoelectric effect, although the photon never actually comes into being within the atom.
- (b) Bragg diffraction: Bragg's famous x-ray experiments consisted of directing an x-ray beam upon a crystal and measuring the scattered photons. Bragg noticed that the beam was only diffracted at certain angles which correspond to different spacing's between the atoms (or crystal planes). The maxima can be found at $2d\sin\theta=m\lambda$, where d is the lattice spacing, θ is the scattering angle, m is the index of the maxima, and λ is the wavelength of x-ray radiation.

- (c) Rutherford scattering: Rutherford found that incident charged particles are scattered by atomic nuclei. There are a number of large-angle scattering events (even back-scattering) which cannot be explained if the atomic charges were distributed in a uniform way through a material. This shows that the charge in an atom is concentrated in a point-like core (the nucleus).

Rutherford scattering equation:

$$\cot\left(\frac{\theta}{2}\right) = \frac{4\pi\epsilon_0 K}{Ze^2} b$$

Where the incident particle has kinetic energy K , the target nucleus has charge Ze^2 and b is the impact parameter.

- (d) The Mössbauer effect: Certain atomic nuclei emit photons in undergoing transitions from “excited” energy states to their “ground” or normal states. These photons constitute gamma rays. When a nucleus emits a photon, it recoils in the opposite direction. If these nuclei are within a crystal, the entire crystal recoils when a gamma-ray photon is emitted instead of the individual atom. This is the Mössbauer effect. The energy of the emitted photons can be shifted in a crystal due to interactions with the crystal field.

- (e) The Stern-Gerlach experiment: In this classic experiment, a beam of silver atoms is directed through a spatially-varying magnetic field. Due to the interaction with the field the beam will be split into two beams, corresponding to different polarizations. This shows the existence of half-integer spin particles.

Solution 12

We have that:

$$H|\psi\rangle = \left[-\frac{\hbar}{2m} \frac{d^2}{dx^2} + V(x) \right] |\psi\rangle = E|\psi\rangle,$$

so that we can substitute $|\psi\rangle$ into this to find $V(x)$ and E . One derivative gives us:

$$\frac{d}{dx}|\psi\rangle = A(-2A^2x) \exp[-a^2x^2],$$

and a second gives us:

$$\frac{d^2}{dx^2}|\psi\rangle = A[(-2a^2) + (-2a^2x)^2] \exp[-a^2x^2] = [-2a^2 + 4a^4x^2]|\psi\rangle.$$

This is a harmonic oscillator and the wave function describes its ground state.