Physics Qualifying Examination

Problems 1–6	Thursday, August 27, 2009	1–5 pm
Problems 7–12	Friday, August 28, 2009	1-5 pm

1. Solve each problem.

2. Start each problem solution on a fresh page. You may use multiple pages per problem.

3. At the top of each solution page put the problem number (1–12) and your Social Security number, but not your name or any other information.

Problem 1

Consider a one-dimensional harmonic oscillator Hamiltonian

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 ,$$

and add the following perturbation

$$H' = -\lambda x .$$

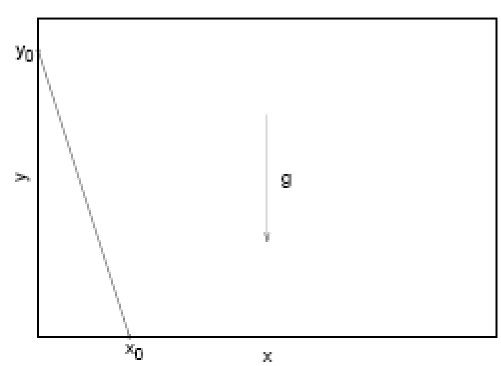
(a) Write the unperturbed Hamiltonian H_0 in terms of raising and lowering operators. What are the energy eigenvalues and level degeneracies?

(b) Write H' in terms of raising and lowering operators. Treating H' as a perturbation, compute the shift in the nth energy level to second order in perturbation theory.

(c) Find the exact energy eigenvalues of the full Hamiltonian, $H_0 + H'$, and compare with your results in (a) and (b).

Hints: $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$, $p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$, $a_+\phi_n = \sqrt{n+1}\phi_{n+1}$ and $a_-\phi_n = \sqrt{n}\phi_{n-1}$. Here $\phi_n(x)$ is the nth excited state of the harmonic oscillator.

Problem 2



The uniform ladder of the figure is of total mass m and leans against a frictionless vertical wall. What is the minimum coefficient of static friction necessary between the ladder and the floor if the ladder is not to slip?

Problem 3

A photon with wavelength 24.8 fm strikes a proton at rest. The photon undergoes Compton scattering, and the scattered photon is seen by an observer in the lab to be emitted at 180° with respect to the direction of the incident photon. The mass of the proton is $M_p = 938$ MeV/c^2, and $hc = 1240$ MeV·fm.

a) What is the energy of the incident photon? What name would typically be given to classify this "type" of photon? Give a very brief explanation for your choice.

b) Using relativistic kinematics, find (i) the wavelength of the scattered photon and (ii) the de Broglie wavelength of the recoiling proton.

c) If we could observe the reaction occurring in the center-of-mass frame instead of the lab frame, what would we then see as the difference between the wavelengths of the incoming and scattered photons? Explain your answer.

Problem 4

A thin fiber of length L is stretched between two supports. The speed of propagation of transverse waves on the fiber is c for both polarizations.

(a) What is the contribution of these modes to the heat capacity of the fiber at low temperatures, assuming $\hbar c/L << k_B T$? (Don't worry about solving the integral to find the dimensionless prefactor)

(b) What is the heat capacity for $\hbar c/L >> k_B T$?

Problem 5

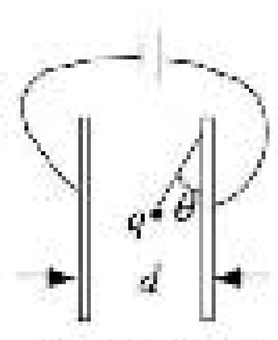
A classical particle of mass m moves in a circular orbit under the influence of a central force $V = -mk/r^n$, where k is a constant.

(a) Write down an effective potential function for this system (centrifugal potential + central potential).

(b) Using the conditions for a circular orbit, solve for the condition on n under small perturbations of the radius.

Problem 6

A parallel-plate capacitor is made of two plates of surface area *A* that are held a distance *d* apart in vacuum. A test charge *q* is suspended from a non-conducting string between the plates as shown. Assume that the plates are very long and that the charge *q* is far from the ends (i.e., ignore edge effects).



(not to scale)

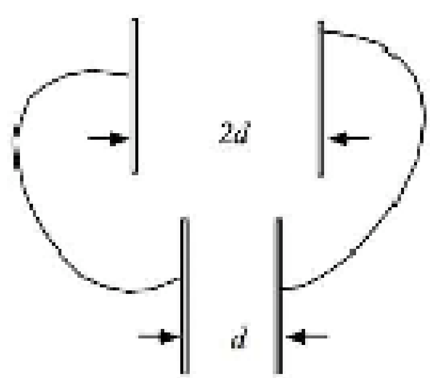
A. The capacitor is attached to a battery as shown. If the distance between the plates is increased from d to 2*d*, state whether any of the following quantities would change, and if so, how.

i. The potential difference *V* across the capacitor.
ii. The charge *Q* on each plate.
iii. The angle θ.

B. Suppose the plate separation is increased from d to 2*d* after the battery is detached. State whether any of your answers to part *A* would be different, and explain how.

C. The distance between the plates is returned to d and the capacitor is again attached to the original battery, which has potential difference V_0 . It is then detached from the battery and connected to a second, initially uncharged capacitor as shown. The plates of the new capacitor have the same area as those of the original capacitor, but the separation between them is 2*d*. Determine the following in terms of *d*, *A*, and V_0 :

i. The potential difference across the original capacitor.
ii. The potential difference across the new capacitor.
iii. The ratio of the charge on the original capacitor to that on the new capacitor.



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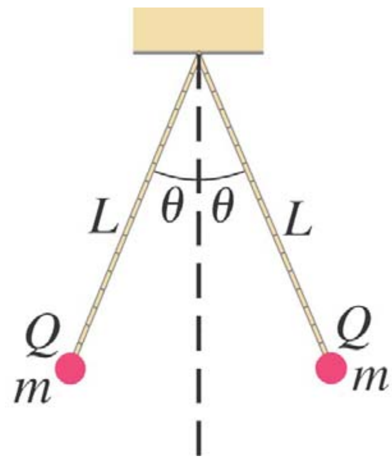
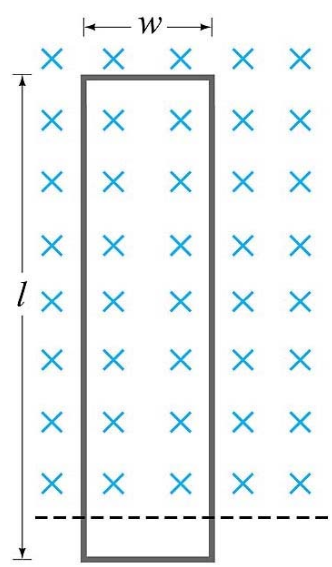
Problem 7

Two small spheres of mass m are suspended from a common point by threads of length L. When each sphere carries a charge q, each thread makes an angle θ with the vertical as shown in the figure. Find an expression for the charge q at equilibrium in terms of L, m, g, θ, and the Coulomb constant k.



Problem 8

The accompanying figure shows a rectangular loop of wire $w = 0.30$m wide and $l = 1.50$m long, in the vertical plane and perpendicular to a uniform magnetic field $B = 0.40$ T, directed into the page as shown. The magnetic field is limited to the region above the dashed line. The portion of the loop not in the magnetic field is 0.10m long. The resistance of the loop is 0.20 Ω and its mass is 0.50 kg. The loop is released from rest at time $t = 0$.



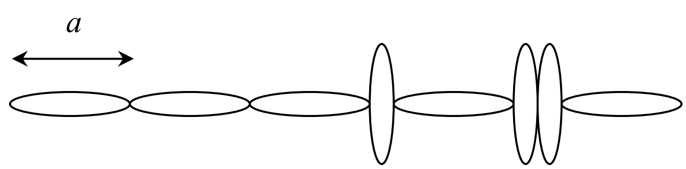
a. What is the magnitude and direction of the induced current when the loop has a downward velocity v?

b. What is the net force acting on the loop?

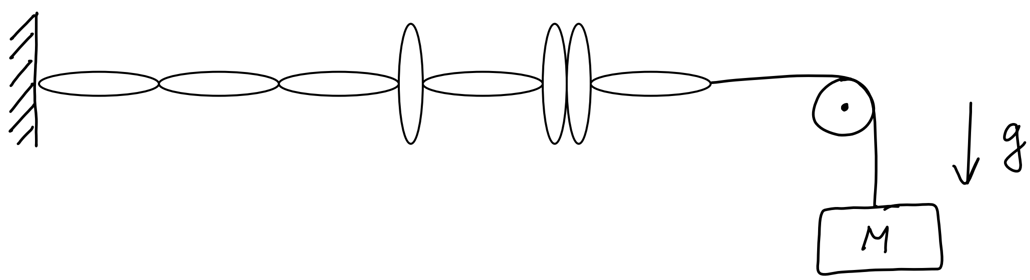
Write the equation of motion of the loop and obtain an expression for the velocity of the loop as a function of time and an expression for the dis-placement of the loop as a function of time. Your answer should include only t as an unknown. All constants should be evaluated. Consider only times short enough that part of the loop is still inside the magnetic field.

Problem 9

A simple model of a rubber band is a one-dimensional (horizontal) chain consisting of N ($N>>1$) linked segments, as shown in the diagram. Each segment has just two possible states: horizontal with length a and vertical , contributing nothing to the length. The segments are linked such that they cannot come apart. The chain is in thermal contact with a reservoir at temperature T.



(a) If there is no energy difference between the two states, what is the average length of the chain?
(b) The chain is now fixed at one end and a weight hung from the other end, supplying a force $F=Mg$ as shown below. Determine the average length of the chain at any temperature T. Find the length in the limits of low and high T.



(c) In which temperature limit is the extension proportional to F (Hooke's law)? Calculate the constant of proportionality.

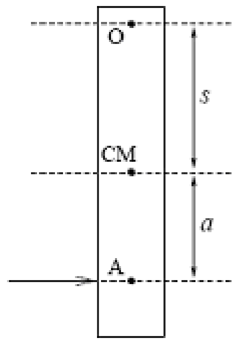
Problem 10

Consider a small spinless non-relativistic particle of mass m moving near the surface of Earth.
Assume that it is dropped from a small height h and bounces elastically.

(a) Using classical mechanics, find the particle's position and momentum as a function of time.
(b) Now using quantum mechanics, determine the Hamiltonian operator for the above problem.
(c) Find the quantum energy spectrum using the WKB approximation.

Problem 11

A uniform rod of mass M and length $2b$ is pivoted at a point O, a distance s above the center of mass. The rod is struck with a rapid impulsive force perpendicular to the rod at a point A, a distance a below the center of mass. The magnitude of the impulse is $P = F\Delta t$. Find the value of a such that there is no reaction at the pivot during the impact. (The moment of inertia of a uniform rod of length L and mass M about an axis through its center perpendicular to its length is $I = ML^2/12$).



Problem 12

Eigenstates of the one-dimensional harmonic oscillator with potential $V(x) = m\omega^2 x^2 / 2$ are given by

$$|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n |0\rangle \quad \text{where} \quad a^\dagger = \sqrt{\frac{M\omega}{2\hbar}}\left(x + \frac{ip}{M\omega}\right)$$

1. Write down the energy eigenvalues E_n.
2. Calculate $\langle m|a|n\rangle$ and $\langle m|a^\dagger|n\rangle$.
3. Calculate $x_{mn} = \langle m|x|n\rangle$.
4. Find the Thomas-Reiche-Kuhn sum rule for $\sum_n (E_n - E_m)|x_{mn}|^2$.