Modern Physics Solutions

Solution 1

To find the fraction of particles which reach the detector, we have to find the time that it takes for particles to reach the detector. Let N_0 be the number of the generated particles and N be the number of particles which reach the detector. The relation between them is given by:

$$N = N_0 \exp\left(-\frac{t}{\tau}\right). \tag{1}$$

Eq.1 is the formula which is used to find the number of particles after time t when their life time is τ. Therefore we have to find t and τ.

The generated particles have a life time $\tau_0 = 100$ ns, in their rest frame. This means that if we "travel" with the particle the particle will decal after 10ns. But this is not the time which is measured in the laboratory. Since the rest frames of the particles move with respect to the lab-frame. According to the special relativity there is dilation of time between the time measured in the lab-frame and the one in the rest frame of the particle. The measured time τ is related to τ_0 by:

$$\tau = \gamma\tau_0 = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{2}$$

where v is the velocity of the particles. Here the velocity of the particles is unknown. To find the velocity we use the information given in the problem. We know the total energy of the particle. The relation between the total energy and the velocity is given by:

$$E^2 = p^2c^2 + m_0^2c^4 \tag{3}$$

but, $p = \gamma m_0 v$

$$E^2 = \gamma^2 m_0^2 c^2 + m_0^2 c^4$$

$$= \left[\gamma^2 v^2 + c^2\right] m_0^2 c^2$$

using the definition of $\gamma = \frac{1}{1-\frac{v^2}{c^2}} m_0^2 c^4$

$$= \gamma^2 \; m_0^2 c^4$$

or $E = \gamma m_0 c^2$ (5)

The total energy is:

$$E = 100 m_0 c^2 \Rightarrow \gamma = 100 \tag{6}$$

Substitute Eq.6 in Eq.2 we obtain:

$$\tau = 10^{-8}\, s. \tag{7}$$

From Eq.6 and the definition of γ we get:

$$v \approx c \tag{8}$$

The time for particles to reach the detector is given by:

$$t = \frac{1}{v} \approx \frac{1}{c} = 2\times 10^{-8}\, s. \tag{9}$$

Substitute Eq.7 and 9 in Eq.1 it yields:

$$\frac{N}{N_0} = e^{-2} \approx 13.5\%. \tag{10}$$

Solution 2

(a) The Auger Effect: An atom with a missing inner electron can lose excitation energy by the Auger effect without emitting an x-ray photon. In the Auger effect an outer-shell electron is ejected from the atom at the same time that another outer-shell electron drops to the incomplete inner shell. Thus the ejected electron carries off the atom's excitation energy instead of a photon doing this. In a sense, the Auger effect represents an internal photoelectric effect, although the photon never actually comes into being within the atom.

(b) Bragg diffraction: Bragg's famous x-ray experiments consisted of directing an x-ray beam upon a crystal and measuring the scattered photons. Bragg noticed that the beam was only diffracted at certain angles which correspond to different spacing's between the atoms (or crystal planes). The maxima can be found at 2dsinθ=mλ, where d is the lattice spacing, theta is the scattering angle, m is the index of the maxima, and lambda is the wavelength of x-ray radiation.

(c) Rutherford scattering: Rutherford found that incident charged particles are scattered by atomic nuclei. There are a number of large-angle scattering events (even back-scattering) which cannot be explained if the atomic charges were distributed in a uniform way through a material. This shows that the charge in an atom is concentrated in a point-like core (the nucleus).
Rutherford scattering equation:

$$\cot\left(\frac{\theta}{2}\right) = \frac{4\pi\varepsilon_0 K}{Ze^2} b$$

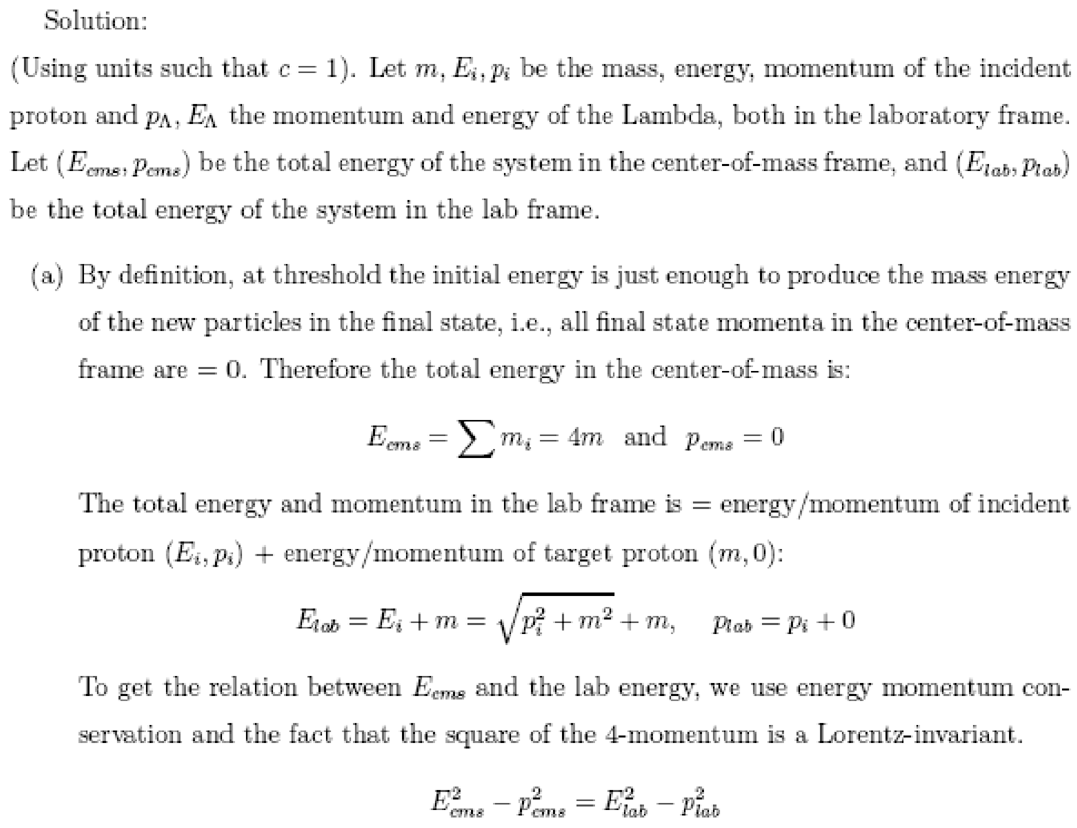
Where the incident particle has kinetic energy K, the target nucleus has charge Ze^2 and b is the impact parameter.

(d) The Mössbauer effect: Certain atomic nuclei emit photons in undergoing transitions from "excited" energy states to their "ground" or normal states. These photons constitute gamma rays. When a nucleus emits a photon, it recoils in the opposite direction. If these nuclei are within a crystal, the entire crystal recoils when a gamma-ray photon is emitted instead of the individual atom. This is the Mössbauer effect.

The energy of the emitted photons can be shifted in a crystal due to interactions with the crystal field.

(e) The Stern-Gerlach experiment: In this classic experiment, a beam of silver atoms is directed through a spatially-varying magnetic field. Due to the interaction with the field the beam will be split into two beams, corresponding to different polarizations. This shows the existence of half-integer spin particles.

Solution 3



Solution:

(Using units such that $c = 1$). Let m, E_i, p_i be the mass, energy, momentum of the incident proton and p_Λ, E_Λ the momentum and energy of the Lambda, both in the laboratory frame. Let (E_{cms}, p_{cms}) be the total energy of the system in the center-of-mass frame, and (E_{lab}, p_{lab}) be the total energy of the system in the lab frame.

(a) By definition, at threshold the initial energy is just enough to produce the mass energy of the new particles in the final state, i.e., all final state momenta in the center-of-mass frame are $= 0$. Therefore the total energy in the center-of-mass is:

$$E_{cms} = \sum m_i = 4m \quad \text{and} \quad p_{cms} = 0$$

The total energy and momentum in the lab frame is = energy/momentum of incident proton (E_i, p_i) + energy/momentum of target proton $(m, 0)$:

$$E_{lab} = E_i + m = \sqrt{p_i^2 + m^2} + m, \qquad p_{lab} = p_i + 0$$

To get the relation between E_{cms} and the lab energy, we use energy momentum conservation and the fact that the square of the 4-momentum is a Lorentz-invariant.

$$E_{cms}^2 - p_{cms}^2 = E_{lab}^2 - p_{lab}^2$$

$E_{cms}^2 - p_{cms}^2 = E_{cms}^2 = (4m)^2$ (since $p_{cms} = 0$), and

$E_{lab}^2 - p_{lab}^2 = E_i^2 + m^2 + 2E_i m - p_i^2 = p_i^2 + 2m^2 + 2E_i m - p_i^2 = 2m(E_i + m)$

We therefore have $(4m)^2 = 2m(E_i + m)$ and thus we get for the energy and momentum of the incident proton at threshold:

$$E_i = 7m$$

and

$$p_i = \sqrt{E_i^2 - m^2} = \sqrt{49m^2 - m^2} = 4m\sqrt{3}$$

(b) Since in the center-of-mass frame all final state particles have 0 momentum, and since by assumption they all have the same rest mass, they also have equal momenta in the lab, i.e. $4p_\Lambda = p_i$ or

$$p_\Lambda = p_i/4 = m\sqrt{3}, \quad E_\Lambda = \sqrt{p_\Lambda^2 + m^2} = 2m.$$

To calculate the mean distance travelled, we need the speed of the Λ. We use the relations

$$E = m\gamma, \quad p = m\beta\gamma,$$

where

$$\beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

The speed of the Lambda v is given by $c \cdot \beta_\Lambda$, where

$$\beta_\Lambda = \frac{p_\Lambda}{E_\Lambda} = \frac{\sqrt{3}}{2}$$

Because of time dilation, the mean lifetime of the Lambdas in the lab $\tau_{lab} = \gamma \cdot \tau$. The distance d travelled by the Lambda in this time is $d = v \cdot \tau_{lab}$,

$$d = (\frac{\sqrt{3}}{2} \cdot 3 \cdot 10^8 \cdot 2 \cdot 2.6 \cdot 10^{-10} m = 7.8\sqrt{3} \cdot 10^{-2} m \approx 13.5cm$$

(c) The decay follows an exponential decay law: the number $N(t)$ not yet decayed at time t is $N(t) = N_0 e^{-t/\tau}$. In other words, the survival probability, i.e. the probability that a particle's time of decay t_{decay} is longer than some given time t_d is

$$P(t_{decay} \geq t_d) = e^{-t_d/\tau}$$

For $t_d = \tau$, this gives $e^{-1} = 1/e \approx 0.368$. Thus the probability that Lambda particle (1) has a lifetime longer than its mean lifetime is $P(1) = 1/e$, and for Lambda (2) it is $P(2) = 1/e$. Since the decays of the two Lambda particles are independent events, the probability for at least one of the two (either of the two) surviving beyond its mean lifetime is

$$P(1 \ or \ 2) = P(1) + P(2) - P(1 \ and \ 2).$$

The reason we have to subtract $P(1 \ and \ 2)$ is to avoid double counting, since $P(1)$ includes those cases where in addition to particle (1) also particle (2) lived beyond its mean time (and *mutatis mutandis* for $P(2)$)
Thus the probability that at least one of the two Lambdas will travel the distance determined in (b) is

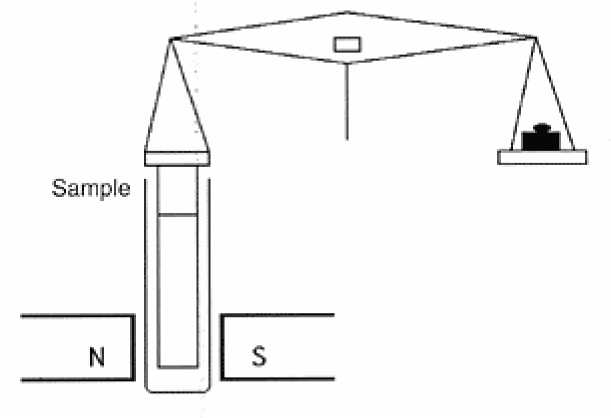
$$P(at \ least \ one) = 2e^{-1} - e^{-2} \approx 0.60$$

Solution 4

(a) Electrical resistivity: ρ, the constant of proportionality that relates $R = \rho\,(L/A)$, where L is the length of a wire, A is the cross-sectional area, and R is the resistance. To measure the resistivity of a conductor, you would measure the length of the wire, the cross-sectional area, and then the resistance through V=IR or R = V/I (V and I are the voltage and current, respectively)

(b) Magnetic susceptibility: $\chi = M/B$, where M is the magnetization, and B is the applied magnetic field. The magnetic susceptibility can be measured in several different ways – the Gouy balance is one method, where you can measure the torque in a magnetic field of a material. The magnet remains stationary while the sample moves, giving an apparent weight loss or gain. Another way is through SQUID magnetometry (Superconducting Quantum Interference Device), which measures changes in the magnetic flux quantum.

A Gouy balance:



(c) Specific heat: the amount of heat required to raise the temperature of either 1 g or 1 mol of a substance by 1 K: $c = (1/m)*\Delta Q/\Delta T$ (m is the mass or molar mass, Q is the amount of heat, T is the temperature). You can measure the specific heat by adding heat to a known amount of substance and then calculating the temperature change. Of course, everything must be well insulated to prevent excess heat loss or gain to the sample.

(d) Thermal conductivity: the constant of proportionality k that relates the heat flow in a certain amount of time to the value of the heat gradient across an object of cross-sectional area A.

$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$$

Here, ΔQ is the amount of heat that flows in time Δt, the rod has cross-sectional area A, and the gradient is $\Delta T/\Delta x$. To measure this, you would need to establish a temperature gradient across a cylindrical object that you wish to measure (of length x and cross-sectional area A), and then you would need to measure the heat flow as a function of time. These experiments are done in thermally isolated containers to prevent heat loss from other sources.

(e) Dielectric constant: this is the factor by which the internal electric field of a material changes when an electrical field is applied: $\kappa=\varepsilon_s/\varepsilon_0$ where ε_s is the static permittivity and ε_0 is a constant. This can be measured through a capacitor, where the capacitance is $C = \kappa\varepsilon_0 A/d$ (A is the cross-sectional area and d is the distance between the plates). In metals, the dielectric constant can be infinite, and so other methods are needed.