

**Qualifying Exam Spring-2007**  
**Classical Mechanics**  
**Problem 1**

A pencil of length  $L$  is held vertically with its point on a desk and then allowed to fall over. Assuming that the point does not slip, find the speed of the eraser just as it strikes the desk. Compare this with the speed that would result from free fall from a height equal to the length of the pencil.

**Qualifying Exam Spring-2007**  
**Electrodynamics**  
**Problem 2**

A static charge distribution produces a spherically radial electric field

$$\vec{E} = A \frac{\exp(-br)}{r^2} \vec{r}$$

where  $A$  and  $b > 0$  are constants.

- 1) What is the charge density  $\rho(r)$ ? Make a sketch of the function.
- 2) What is the total charge  $Q$ ?

**Qualifying Exam Spring-2007**  
**Quantum Mechanics**  
**Problem 3**

Consider two solutions to the one-dimensional time-independent Schrödinger equation with same energy  $E$ :  $\psi_1(x)$  and  $\psi_2(x)$ .

- (a) Prove that regardless of the potential function  $V(x)$ ,

$$\psi_1 \frac{d}{dx} \psi_2 - \psi_2 \frac{d}{dx} \psi_1 = C$$

where  $C$  is a constant.

- (b) Show that if  $\psi_1(x)$  and  $\psi_2(x)$  are bound state solutions, then  $C = 0$ .

From this, show that  $\psi_1 = \gamma \psi_2$  for some constant  $\gamma$ , thus proving that there are no degenerate bound state solutions to the one-dimensional time-independent Schrödinger equation.

- (c) Why does this theorem fail for continuum eigenstates (i.e. unbound states)? Give a specific example of degenerate continuum eigenstates in one dimension.

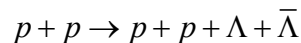
**Qualifying Exam Spring 2007**  
**Thermodynamics/Statistical Mechanics**  
**Problem 4**

A certain amount of water of heat capacity  $C$  is at a temperature of  $0^\circ\text{C}$ . It is placed in contact with a heat reservoir at  $100^\circ\text{C}$  and the two come into thermal equilibrium.

- (a) What is the entropy change of the universe.  
(b) The process is now divided into two stages: first the water is brought into contact with a heat reservoir at  $50^\circ\text{C}$  and comes into thermal equilibrium; then it is placed in contact with the heat reservoir at  $100^\circ\text{C}$ . What is the entropy change of the universe.  
(c) If we were to continue this subdivision into an infinite number of heat baths, what would the entropy change of the universe be?

**Qualifying Exam Spring 2007**  
**Modern Physics**  
**Problem 5**

Consider the reaction

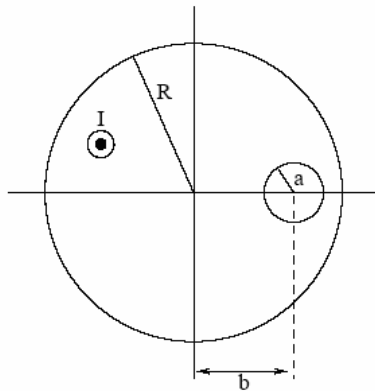


Assume that the masses of proton and Lambda are  $1 \text{ GeV}/c^2$ .

- (a) Consider a laboratory frame in which one of the two initial state protons is at rest. What is the "threshold" energy, i.e. the minimum energy that the incident proton must have for the reaction to be kinematically possible? (Hint: What does this mean in terms of final state momenta in the center-of-mass frame?)  
(b) For the reaction at threshold as described in part (a), what is the mean distance that the Lambda travels before it decays? (the mean lifetime of the Lambda in its rest frame is  $\tau \approx 2.6 \cdot 10^{-10} \text{ s}$ )  
(c) What is the probability that at least one of the two Lambdas travels the distance determined in (b)?

**Qualifying Exam Spring 2007**  
**Electrodynamics**  
**Problem 6**

A very long, straight conductor with a circular cross section of radius  $R$  carries a current  $I$ . Inside the conductor is a cylindrical hole of radius  $a$  whose axis is parallel to the axis of the conductor but offset a distance  $b$  from the axis of the conductor. The current  $I$  is uniformly distributed across the cross section of the conductor and is directed out of the page. Find the magnetic field everywhere outside the conductor.



**Qualifying Exam Spring 2007**  
**Classical Mechanics**  
**Problem 7**

Consider the Earth as a frame rotating about its axis with frequency  $\omega$ . Particles are then subject to the Coriolis force given by  $\vec{F} = -2m(\vec{\omega} \times \vec{v}_r)$ , where  $\vec{v}_r$  is the velocity of the particle relative to the frame and  $m$  is the mass of the particle. Choose the  $z$ -axis along the upwards vertical direction, and the  $y$ -axis pointing North, and the  $x$ -axis to the East. Assume you are in the Northern hemisphere at a colatitude  $\theta$  (angle of the  $z$ -axis with respect to  $\vec{\omega}$ ).

- (a) a particle moves under the influence of gravity and the Coriolis force. Write the equations of motion in the  $x$ ,  $y$ , and  $z$  directions. Use the approximation that the component of velocity in the  $z$  direction is much larger than the components in the  $x$  and  $y$  directions, i.e.  $|v_z| \gg |v_x|, |v_y|$ .
- (b) the particle is dropped at rest from a height  $h$  above the ground, and it arrives at the ground with a speed  $v_0 = \sqrt{2gh}$ . Find the magnitude and direction of the Coriolis deflection.

- (c) the particle is now thrown vertically upward with an initial speed  $v_0$  so that it reaches the maximum height  $h$  [the same height as in part (b)], and then falls back to the ground. find the magnitude and direction of the Coriolis deflection.
- (d) Compare your results for parts (b) and (c).

**Qualifying Exam Spring 2007**  
**Quantum Mechanics**  
**Problem 8**

Consider a quantum mechanical system with two states,  $|\alpha\rangle$  and  $|\beta\rangle$ . In this orthonormal basis of states the Hamiltonian is given by the matrix

$$\begin{pmatrix} W & V \\ V & -W \end{pmatrix}.$$

- (a) Obtain the exact energy eigenvalues.
- (b) Consider the Hamiltonian as  $H = H_w + H_v$ , where

$$H_w = \begin{pmatrix} W & 0 \\ 0 & -W \end{pmatrix}, \quad H_v = \begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}$$

and assuming that  $|V| \ll |W|$  obtain the energy eigenvalues to second order in perturbation theory in  $V$ .

- (c) Compare your results in (a) and (b) and verify they agree to second order in  $V$ .

**Qualifying Exam Spring 2007**  
**Modern Physics**  
**Problem 9**

Define the following properties of a solid, and explain how each can be measured

- (a) electrical resistivity  
 (b) magnetic susceptibility  
 (c) specific heat  
 (d) thermal conductivity  
 (e) dielectric constant

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**Electrodynamics**  
**Problem 10**

A thin circular copper ring rotates about an axis which is perpendicular to a uniform magnetic field  $H_0$ . The rotation axis is a diameter of the ring. The rotation of the ring slows down because the rotational energy is dissipated into Joule heating.

(a) Estimate the time  $\tau$  it takes for the angular frequency to decrease to  $1/e$  of its original value. In the calculation assume that the angular frequency varies with time as  $d\omega/dt = -\omega/\tau$ . Assume that the initial angular frequency of rotation,  $\omega_0$ , is much larger than  $1/\tau$ . Assume as well that you can average over times of the order of  $1/\omega_0$ .

Copper has conductivity  $\sigma = 5 \times 10^{17} \text{ s}^{-1}$  (cgs units) and density  $8.9 \text{ gcm}^{-3}$ . Assume that  $H_0$  is 200G and the radius of the ring is 10 cm. Note that  $\text{G}^2 = \text{erg/cm}^3$  has the units of an energy density.

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**Quantum Mechanics**  
**Problem 11**

Consider the hyperfine splitting of the ground state of Hydrogen in a uniform magnetic field  $B$ . Assume the Zeeman energy shift is of the same order of magnitude as the hyperfine shift. The perturbation Hamiltonian is

$$V = A \vec{s}_e \cdot \vec{s}_p - \vec{\mu}_e \cdot \vec{B}$$

where  $\vec{\mu}_e = (e/m_e c) \vec{s}_e$ . The sub-indices  $e$  and  $p$  refer to the electron and proton, respectively. Using degenerate perturbation theory, find the energy shifts of all four of the  $n = 1, l = 0$  levels. You may ignore the  $-\vec{\mu}_p \cdot \vec{B}$  term.

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**Thermodynamics/Statistical Mechanics**  
**Problem 12**

Consider a 2-level system with energy states  $\varepsilon$  and  $\varepsilon + \Delta$  ( $\Delta \geq 0$ ). Compute the partition function and the free energy. Derive an expression for the specific heat  $C(T)$ . What are the low-T and high-T limits of this expression? Sketch your result.

