Physics Qualifying Examination

Problems 1–6	Thursday, January 10, 2008	1–5 pm
Problems 7-12	Friday, January 11, 2008	1-5 pm

1. Solve each problem.

2. Start each problem solution on a fresh page. You may use multiple pages per problem.

3. Turn in at least one page per problem, even if you give no solution. At the top of each solution page put the problem number (1–12) and your Social Security number, but not your name or any other information.

1. A parallel-plate capacitor has circular disk electrodes of radius a that are separated by a distance d ($d << a$). A resistor of resistance R connects the plates along their common axis. At $t = 0$, the capacitor is suddenly given charge Q_0. Neglect all edge effects.

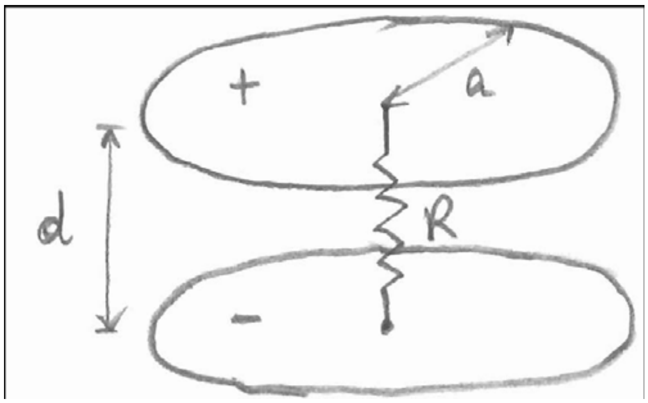
(a) What is the capacitance of the capacitor?

(b) Obtain the time-dependent charge on the capacitor plates.

(c) Find the electric field between the plates.

(d) Obtain the magnetic field between the plates as a function of position and time.

(e) Derive an expression for the Poynting vector $\vec{S}$ as a function of position between the plates.



2. A hydrogen atom is placed in a uniform electric field $\vec{E}(t)$ in the z-direction. $\vec{E}(t)$ is 0 for $t < 0$ and $\vec{E}(t) = \vec{E}_0 e^{-t/\tau}$ for $t > 0$. The atom is initially in the ground state.

(a) Find the probability for the atom to have made a transition to the 2s state as $t \to \infty$.

(b) Find the probability for the atom to have made a transition to one 2p state as $t \to \infty$.

(c) Assuming that $\vec{E}_0$ is small, how small should it be for the calculation to be valid?

The following radial wave functions of hydrogen may be useful to solve the problem:

$$R_{10} = \frac{1}{a^{3/2}} 2 \exp(-r/a)$$

$$R_{20} = \frac{1}{(2a)^{3/2}} (2 - r/a) \exp(-r/2a)$$

$$R_{21} = \frac{1}{(2a)^{3/2}} (r/\sqrt{3}a) \exp(-r/2a)$$

where a is Bohr's radius. The relevant spherical harmonics are $Y_{00} = 1/\sqrt{4\pi}$ and $Y_{10} = \sqrt{3/4\pi} \cos\theta$.

3. A bucket of liquid (density ρ) is set spinning about its vertical symmetry axis with angular velocity ω in the presence of Earth's gravitational field. Determine the shape of the liquid's surface in the bucket.
Solve this problem in steps:

a) Draw a free-body diagram for an element of the liquid at the surface.
b) Determine the angle between normal to the surface and the vertical direction as a function of distance from the axis of rotation.
c) Find a function that describes the shape of the surface.

4. Write down the following named electrodynamics equations (in any system of units that you desire, but algebraically indicate the unit-defining constants such as speed of light c, where appropriate). Identify/name all quantities that appear in the equations. A small figure or picture defining the terms and context can be useful.

(a) The Biot-Savart Law
(b) Ampere's Law (circle the Maxwell displacement current)
(c) Kirchhoff's Rule for current junctions
(d) Faraday's law (of induction)
(e) The Poynting vector

5. Spin waves (magnons) in a ferromagnetic solid have a dispersion relation of the form:

$$\omega = A|k|^2$$

Treating the magnons as elementary excitations, calculate the temperature dependence of the heat capacity of the spin system at low temperatures. You might find the following integral to be useful:

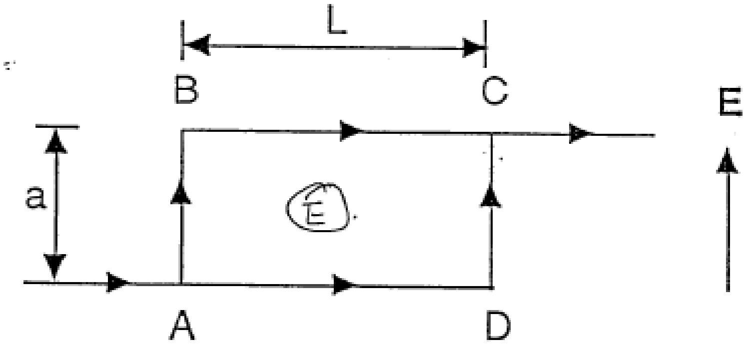
$$\int_0^\infty \frac{x^{3/2}dx}{e^x - 1} = \text{finite constant}$$

6.

(a) The molten metal in a furnace appears to emit predominantly blue light. Is the temperature of the metal closest to (I) 3 K, (II) 3 x 10^4 K, or (III) 3 x 10^6 K?

(b) Estimate the ground state energy of a harmonic oscillator using the uncertainty principle.

(c) In an electron interferometer, an electron is split into two paths ABC and ADC in the figure below and recombined. The rectangular paths have the dimensions AB = DC = a and AD = BC = L. The whole rectangle is placed in a uniform electric field along the direction AB or DC. If interference fringes are produced as the electric field strength is varied, find the interference fringes' separation as the change of the electric field in terms of the electron kinetic energy, its mass and charge, the dimensions of the paths and any universal constants.



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7. Light of wavelength 300 nm strikes a metal plate, producing photo-electrons that move with speed of 0.002*c*.
(a) What is the work function of the metal?
(b) What is the critical wavelength of the metal?
(c) What is the significance of this critical wavelength?

8. A boy standing on a ladder drops marbles of mass *M* from a height *H*. He tries to hit a point on the ground. Show that even if he is very careful, the marbles are going to miss the point by and average distance Δx which is proportional to

$$\left(\frac{\hbar}{M}\right)^{1/2}\left(\frac{H}{g}\right)^{1/4}$$

where *g* is the gravitational acceleration. How large is the average distance for *M* = 1g and *H* = 2m? Compare this distance to the radius of an atom and the radius of a nucleus. *Hint*: Use Heisenberg's uncertainty principle.

9. Consider a one-dimensional quantum Harmonic oscillator with energy levels

$$E_n = (n + \tfrac{1}{2})\hbar\omega$$

(a) What is the average energy of this oscillator at temperature *T*?
(b) What is the specific heat of this oscillator?
(c) Give expressions for the specific heat appropriate for the high ($k_B T \gg \hbar\omega$) and low ($k_B T \ll \hbar\omega$) limits.

10.
(a) The ground state wave function for the electron in a hydrogen-like ion with nuclear charge +*Ze* has the form

$$\psi(\vec{r},t) = Ae^{-\alpha r}e^{-iEt/\hbar}.$$

By plugging this into the time-dependent Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{r}\right)\psi = -i\hbar\frac{\partial}{\partial t}\psi$$

determine α and *E* in terms of $\hbar$, *m*, *Z* and *e*. Then normalize the wave function to determine *A* in terms of the same constants.

(b) Consider a He^+ ion which consists of a single electron orbiting a nucleus of charge +2*e*. If this nucleus absorbs a positron the nuclear charge will suddenly become +3*e* (i.e. the ion will become a Li^{+2} ion). If the electron in the helium ion was in its ground state before this interaction, what is the probability that immediately after this interaction the electron in the lithium ion will also be in its ground state?

11. A uniform bowling ball with mass M and radius R is hit by a stick and acquires an initial speed v_0 but no angular velocity (so $\omega_0 = 0$) at $t = 0$. The coefficient of the kinetic friction between the ball and the horizontal surface on which it is moving is μ and the acceleration due to gravity is g. *Hint:* $\int \sin^3\theta d\theta = -\frac{1}{3}\cos\theta(\sin^2\theta + 2)$.

a) Calculate the moment of inertia of the ball about an axis that passes through the center of the ball.

b) Calculate the time t_1 after which the ball rolls without slipping on the surface.

c) Calculate the work done by friction between $t = 0$ and $t = t_1$.

12. The two rails of a superconducting track are separated by a distance d. A conductor can slide along the track. The conductor, initially at rest, is pulled to the right by a constant force F. The friction between the conductor and the track is directly proportional to its velocity with a proportionality constant α. The portion of the conductor between the rails has a resistance of R. The entire setup is in a uniform magnetic field $\vec{B}$ as shown in the figure below. The field $\vec{B}$ points *into* the page.

a) What is the direction of the induced current in the conductor?
b) Determine the magnitude of the velocity of the conductor as a function of time.
c) Determine the magnitude of the induced current as a function of time.
d) Determine the terminal velocity of the conductor.