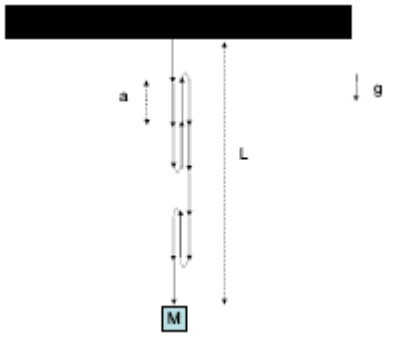
Thermodynamic/Statistical Mechanics
Problem 1

A simple statistical mechanics model of a rubber band consists of *N* connected rigid segments, each of length *a*, which can either point up or down. The total length L of the rubber band is then:

L=a[(# of segments pointing down)-(# of segments pointing up)].

One end of this rubber band is fixed, and a mass M is attached to the other end so that the rubber band hangs vertically (see figure).



(a) Find an expression for the entropy S of this rubber band as a function of its length L. (Hint: Determine the number of microstates Ω, equal to the number of distinct ways the rigid segments can point up or down, when the rubber band has length L, and use the fact that $S = k_B \ln \Omega$, where k_B is Boltzmann's constant).

(b) Find an expression for the free energy of this rubber band as a function of L and the temperature T. Recall that the free energy is F(L,T)= E(L)-TS(L), where, in this case, E(L) is the gravitational potential energy of the mass M (you may ignore the mass of the rubber band) and S(L) is the entropy you found in part (a).

(c) Minimize the free energy with respect to L to obtain the equilibrium length of the rubber band as a function of T, M, g, a and k_B. (Hint: Work in the limit of large N and use Stirling's approximation: $\ln N! \cong N \ln N - N.$)

(d) If the temperature is increased, does the rubber band get longer or shorter?

Thermodynamics/ Statistical Mechanics
Problem 2

A system of N bosons in two dimensions has an energy-momentum relationship:

$$E = cp^{3/2}$$

and density n=N/A (A is the area).

(a) What is Bose-Einstein condensation?

(b) Show that at low temperatures, the system will Bose condense, and that the Bose condensation temperature $T_C \sim n^{\alpha}$. Find α.

(c) Show that the entropy below T_C goes like $S \sim T^{\beta}$, and the "pressure" (equivalent in two dimensions) goes as $P \sim T^{\gamma}$. Find β and γ.
Hint: the differential of the grand potential is:

$$d\theta = -SdT - PdA + nd\mu .$$

Thermodynamics/Statistical Mechanics
Problem 3

A certain amount of water of heat capacity *C* is at a temperature of 0°C. It is placed in contact with a heat reservoir at 100°C and the two come into thermal equilibrium.

(a) What is the entropy change of the universe.

(b) The process is now divided into two stages: first the water is brought into contact with a heat reservoir at 50°C and comes into thermal equilibrium; then it is placed in contact with the heat reservoir at 100°C. What is the entropy change of the universe.

(c) If we were to continue this subdivision into an infinite number of heat baths, what would the entropy change of the universe be?

Thermodynamics/Statistical Mechanics
Problem 4

Consider a 2-level system with energy states ε and ε + Δ (Δ ≥ 0). Compute the partition function and the free energy. Derive an expression for the specific heat C(T). What are the low-T and high-T limits of this expression? Sketch your result.