Physics Qualifying Examination

Problems 1–6	Thursday, August 28, 2008	1–5 pm
Problems 7-12	Friday, August 29, 2008	1-5 pm

1. Solve each problem.

2. Start each problem solution on a fresh page. You may use multiple pages per problem.

3. At the top of each solution page put the problem number (1–12) and your Social Security number, but not your name or any other information.

Problem 1

Consider an ideal monatomic gas consisting of N particles each with mass m. The gas is first expanded from volume V_1 to volume V_2 at constant pressure so that $P_1 = P_2$. The gas is then further expanded isothermally to a final volume V_3 with pressure P_3. The corresponding temperatures are T_1, T_2 and T_3, with $T_2 = T_3$.

(a) Using classical thermodynamics compute the total entropy change and express it in terms of V_1, V_3, T_1, T_3. (Recall that the heat capacity of an ideal monatomic gas at constant pressure is $C_P = \frac{5}{2} N k_B$, where k_B is Boltzmann's constant).

(b) Using the partition function of the ideal classical monatomic gas, calculate the Helmholtz free energy of the system as a function of particles N, the volume V, and the temperature T.

(c) Using the Helmholtz free energy obtained in part (b), obtain an expression for the entropy of the system. Use this expression to compute the total entropy change of the gas and compare your result to that obtained in part (a).

Problem 2

A particle is in the ground state of a one-dimensional infinite square well of width L. The box undergoes a sudden expansion $L \to \alpha L$, with $\alpha > 1$.

(a) What is the probability that the particle will be in the ground state of the expanded well?

(b) Find the behavior of the probability obtained in part (a) in the limit $\alpha \gg 1$ and $\alpha \to 1$.

Problem 3

A vessel holds 2 μg of tritium. The half-life of tritium is $t_{1/2}({}^3_1H) = 12.3$ y, and its mass is $m({}^3_1H) = 3.02\ \text{u} = 5.01 \times 10^{-27}$ kg.

(a) What is the initial decay rate of tritium?

(b) How much time will elapse before the amount of tritium falls to 1% of its initial value?

Problem 4

Consider an electron in a hydrogen atom that has the following wave function at a particular time, $t = 0$:

$$|\psi(0)\rangle = A(|100\rangle + 2i|210\rangle + 2|322\rangle).$$

Here, each of the individual eigenvector terms are denoted by their quantum numbers N (principal), L (angular momentum), and M (angular momentum projection) as $|NLM\rangle$.

(a) Calculate the value of the normalization constant A.

(b) Find the expectation value of the energy of this electron at $t = 0$. Express your answer in units of eV.

(c) If a measurement of the z-projection of the electron's orbital angular momentum is made at $t = 0$, then with what probability are the results 0, $\hbar$, $2\hbar$, $3\hbar$ obtained?

(d) Write the expression for the wave function $|\psi(t)\rangle$ at any time t after $t = 0$.

Problem 5

A classical particle of mass m moves in one dimension and is subjected to a constant external force $F = F_0$ when $x < 0$ and $F = -F_0$ when $x > 0$. F_0 is a positive constant. Assume initial condition $x(0) = a$ and $\dot{x}(0) = 0$.

(a) Construct the mathematical form of the phase space orbit of the motion $\dot{x}(x)$.

(b) Calculate the work done by the external force field during one full period of the orbit.

(c) Calculate the period of the motion.

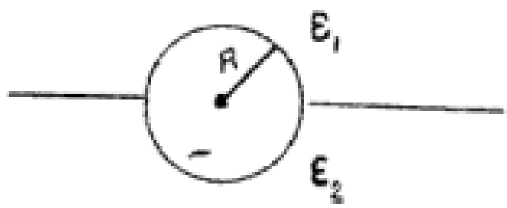
Problem 6

The center of a conducting sphere of radius R resides on a plane that is the interface between two large dielectric slabs (see figure below). The upper slab is characterized by the dielectric constant ε_1 and the lower by the dielectric constant ε_2, and both slabs are infinitely deep. The sphere has a potential V, and we assume the potential is zero at infinity.

(a) What are the field $\vec{E}$ and induction $\vec{D}$ everywhere in space outside the sphere, and what are the boundary conditions for these fields at the interface surface?

(b) What is the total charge Q on the sphere?

(c) What is the distribution of bound charges?



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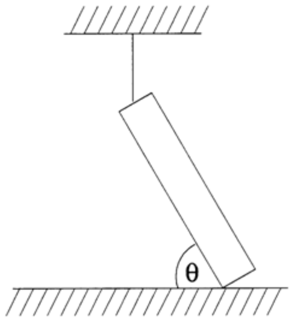
Problem 7

A uniform rod of mass m and length L is suspended on one end by a string, while the other end rests on the ground, such that it forms an angle θ with the horizontal.



a) What is the tension in the string?

b) Now imagine the string is cut, and the rod starts to fall. Calculate the vertical component of the acceleration of the center of the mass of the rod, at the moment $t = 0$ immediately following the cut. Ignore the friction with the ground.

c) What is the force F_N that the rod exerts on the ground at time $t = 0$, immediately after the cut? (*Hint*: It is different than before the string was cut.)

Problem 8

A spaceship is sent from earth to another solar system which is at a distance of 20 light years from the earth. The spaceship is rapidly accelerated to a velocity (relative to the earth) of $0.99c$ toward its objective. Upon reaching the distant solar system, the crew takes some pictures as the ship is rapidly accelerated to $0.99c$ heading back toward the earth. The pictures are immediately transmitted by radio to those waiting on earth. Using special relativity and ignoring the relativistic effects of the periods of acceleration and deceleration:

(a) How long does the round trip take according to people waiting on earth?

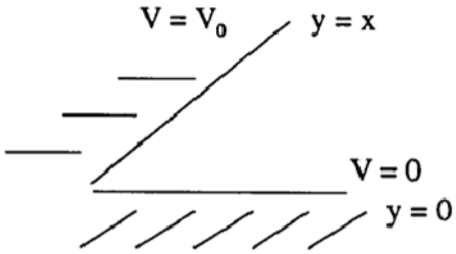
(b) How long does the round trip take for people on the spaceship?

(c) After how many years since takeoff do the people on earth see the first pictures from the distant solar system?

In those pictures, they see the wonders of the distant solar system as well as the clocks on the spaceship. What time do these clocks read (in years)? Show that this is consistent with the Doppler shifted clock frequencies.

Problem 9

Two infinite conducting sheets are placed so that one is in the half plane $y = 0, x > 0$ and the other is in the half plane $y = x, x > 0$. The two sheets are electrically insulated from each other. The sheet at $y = 0$ is kept at zero potential, the sheet at $y = x$ is kept at a potential V_0.



a) Solve for the potential between sheets.

b) Find electric field components in Cartesian coordinates.
Hint: Laplacian in cylindrical coordinates is:

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Problem 10

Consider a particle of mass m that is constrained to move on a sphere of radius R in three dimensions. Use spherical coordinates.

(a) Write the Hamiltonian for such a system in terms of the angular momentum operator and constants. What are the energy eigenvalues and level degeneracies?

(b) Suppose the particle has electric charge q and suppose there is a uniform electric field $\vec{E} = \mathcal{E}\hat{z}$. Write the Hamiltonian for the particle. Using perturbation theory, find the shift in energy to first order in $\mathcal{E}$ for all states.

(c) Now focus on the ground state. To second order in $\mathcal{E}$, do you expect the energy to increase or decrease?

(d) Calculate the second order perturbation correction to the ground state energy.

(e) Obtain the induced electric dipole moment, $\vec{P} = \alpha\vec{E}$, where α is the polarizability. Determine α.

Hint: $Y_{00} = \sqrt{\frac{1}{4\pi}}$, $Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$.

Problem 11

A magnet consists of N >> 1 noninteracting spins which can be in any of three states: $\sigma = 1, 0,$ and -1. Each of these spins is described by a Hamiltonian:

$$E_0\,\sigma^2 - \mu_0\,H\,\sigma$$

where H is the applied magnetic field.

(a) Write the partition function for this system

(b) Calculate the Helmholtz free energy

(c) Calculate the entropy per spin, S/N in the limit as $T \to \infty$

(d) Does the specific heat per spin vanish, or tend to a constant as $T \to \infty$? Explain your answer.

Problem 12

In a region of space there is a uniform magnetic field $\vec{B} = B_0\hat{y}$. An uncharged copper sphere of radius R moves at constant velocity $\vec{v} = v_0\hat{x}$ through the magnetic field. Assume that copper is a perfect conductor and that v_0 is much smaller than c. Assume that the origin of your coordinate system is at the center of the sphere. Use spherical coordinates.

(a) What are the forces acting on the free electrons in the sphere? What is the total force on the electrons? Determine the magnitude and direction of the electric field inside the sphere.

(b) Use your result in (a) to obtain the electrostatic potential inside the sphere.

(c) What is the θ and r dependence of the electrostatic potential outside the sphere? Determine all coefficients by matching the potential inside and outside at the boundary of the sphere.

(d) Obtain the electric field outside the sphere.

Hint: Recall that $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$

(e) Is it necessary to exert a force on the sphere to maintain its motion?