Physics Qualifying Examination

Problems 1–6	Thursday, August 30, 2007	1–5 pm
Problems 7-12	Friday, August 31, 2007	1-5 pm

1. Solve each problem.

2. Start each problem solution on a fresh page. You may use multiple pages per problem.

3. At the top of each solution page put the problem number (1–12) and your Social Security number, but not your name or any other information.

Problem 1

Consider an ideal gas of N electrons that move freely in 2D, and are confined to an area A.

(a) Estimate the temperature T_0 below which quantum mechanical considerations are important.

(b) Calculate the Fermi energy assuming the system is at the temperature T = 0 K. Compare the Fermi energy to $k_B T_0$, where T_0 is calculated above.

(c) What is the physical interpretation of the Fermi energy for a given system?

Problem 2

Give approximate numerical values for the following (be sure to give units):

(a) The binding energy of a nucleon in a typical nucleus.

(b) The binding energy of an electron in a hydrogen atom.

(c) The lifetime of a free neutron.

(d) The specific heat of Cu at room temperature (assumed greater than the Debye temp.)

(e) The energy of a photon of light within the visible part of the spectrum.

Problem 3

3. A cylindrical block of wood of density ρ_w, radius R, and height h is partially immersed in a liquid of density ρ_l and then released, as shown in the figure.

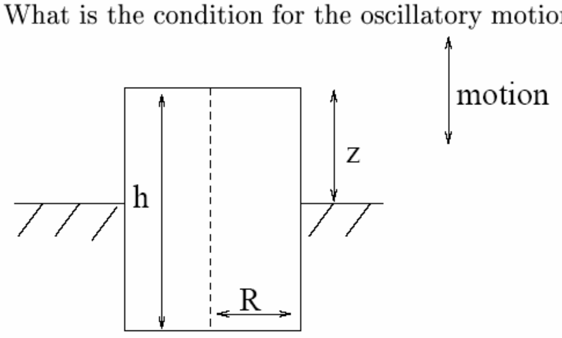
a) What is the condition for the block to keep afloat?

b) What is the equilibrium height of the block above the water level z_{eq}?

c) If the block was initially slightly raised, so that $z(t=0) > z_{\text{eq}}$, and then released, calculate $z(t)$ assuming no friction.

d) Now assume that the liquid is very viscous, and that the viscous force is proportional to the velovity, as given by $\mathbf{F}_v = -b\mathbf{v}$. How is the motion of the block modified?

e) What is the condition for the oscillatory motion to be over-damped?



Problem 4

A positive charge is uniformly distributed throughout a very long cylindrical volume of radius R. The charge per unit volume is ρ.

(a) Find the electric field $\vec{E}$ everywhere as a function of the distance r from the axis of the cylinder.

(b) Find the electric potential V everywhere as a function of r. Define $V = 0$ at the surface of the cylinder.

Sketch E and V as functions of r, from $r = 0$ to $r = 3R$, showing the values of each at $r = 0$, R, and $3R$.

Problem 5

One mole of an ideal gas undergoes a reversible isothermal expansion from volume V to volume $2V$.

(a) What is the change in entropy of the gas?

(b) What is the change in entropy of the universe?

Now suppose the same expansion takes place as a free expansion.

(c) What is the change in entropy of the gas?

(d) What is the change in entropy of the universe?

Problem 6

A quantum particle of mass m moving in one dimension is confined by the wedge potential

$$\begin{aligned} V(x) &= \infty \quad \text{for } x < 0 \\ &= \kappa x \quad \text{for } x \geq 0, \end{aligned}$$

where $\kappa > 0$. Consider the following variational wave function for the ground state of this particle,

$$\begin{aligned} \psi_a &= 0 \quad \text{for } x < 0, \\ &= Axe^{-\alpha x} \quad \text{for } x \geq 0, \end{aligned}$$

with variational parameter α.

(a) Normalize $\psi_a(x)$ and obtain the normalization constant A as a function of α.

(b) Obtain the expectation value of the kinetic energy in the state $\psi_a(x)$, $\langle \psi_a | T | \psi_a \rangle$ where $T = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$.

(c) Obtain the expectation value of the potential energy in the state $\psi_a(x)$, $\langle \psi_a | V | \psi_a \rangle$.

(d) Minimize the total energy $E = \langle \psi_a | T + V | \psi_a \rangle$ with respect to α to obtain a variational upper bound on the ground state energy of the particle.

Physics Qualifying Examination

Problems 1–6	Thursday, August 30, 2007	1–5 pm
Problems 7-12	Friday, August 31, 2007	1-5 pm

1. Solve each problem.

2. Start each problem solution on a fresh page. You may use multiple pages per problem.

3. At the top of each solution page put the problem number (1–12) and your Social Security number, but not your name or any other information.

Problem 7

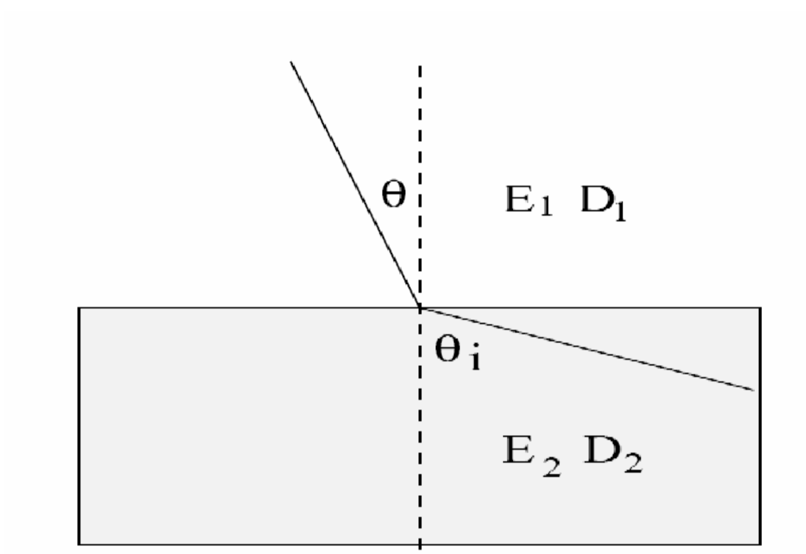
A uniform spherical ball of mass M and radius R is set rotating about a horizontal axis with angular speed ω_0 and is placed gently on the floor. The initial center–of–mass velocity of the ball is zero. If the coefficient of sliding friction between the balls and the floor is μ, find the speed of the center of mass of the ball when it begins to roll without slipping. (The moment of inertia of the ball about an axis passing through it s center of mass is $I = \frac{2}{5}MR^2$.)

Problem 8

A large slab of dielectric material with a plane surface is inserted into an electric field $\vec{E}$, such that $\vec{E}$ is at an angle θ with the normal to the surface of the dielectric. The dielectric constant of the dielectric material is $\varepsilon = \kappa\varepsilon_0$.

(a) find the magnitude of $\vec{E}$ and the magnitude of the displacement vector $\vec{D}$ inside the dielectric.

(b) Find a relationship between the dielectric constant and the angle $\vec{E}$ and $\vec{D}$ make with the normal inside the dielectric. Sketch E and D inside the dielectric assuming $\varepsilon = 3\varepsilon_0$ and $\theta = 30°$.



Problem 9

A dielectric sphere of radius R with uniform dielectric constant ϵ has an azimuthally symmetric charge density $\sigma = \sigma_0 \cos\theta$ placed on its surface. Outside the sphere is vacuum.

(a) Obtain the electrostatic potential inside the sphere, ϕ_{in}.

(b) Obtain the electrostatic potential outside the sphere, ϕ_{out}.

(c) What is the electric field inside the sphere?

Problem 10

Consider a two-dimensional infinite potential well of width L defined by the potential

$$V(x,y) = \begin{cases} 0 & \text{if } 0 \le x \le L \text{ and } 0 \le y \le L \\ \infty & \text{otherwise.} \end{cases}$$

Assume that the well contains one particle of mass m, i.e. $H = (p_x^2 + p_y^2)/2m + V(x,y)$.

(a) Obtain the ground state wave function(s) and energy. Is the ground state degenerate?

(b) Obtain the wave function(s) and energy of the first excited state. Is the first excited state degenerate?

Assume now that a perturbation of the form

$$H' = g \sin(2x\pi/L)\sin(2y\pi/L)$$

is added to H.

(c) Calculate the first order perturbation to the ground state energy eigenvalue.

(d) Calculate the first order perturbation to the first excited state energy eigenvalue.

Problem 11

Neutron scattering is often done by cooling fast neutrons, which are among the fission products from a nuclear reactor, by thermalizing them in a moderator such as solid deuterium oxide (D_2O ice) held at some constant cryogenic temperature T. The average kinetic energy of neutrons from such a source is

$$E = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT,$$

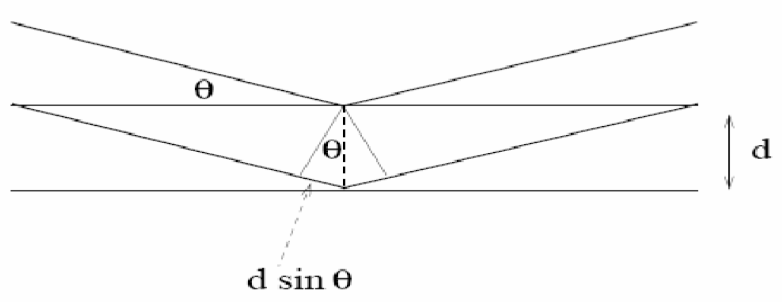
where m is the neutron mass, $\langle v^2 \rangle$ is the mean-square velocity, and k is the Boltzmann constant. The neutron mass is $m_n = 1.68 \times 10^{-27}$ kg.

(a) What is the de Broglie wavelength λ_n for neutrons in terms of their energy, mass, and Planck's constant?

(b) What is the de Broglie wavelength for neutrons with the average kinetic energy in terms of the temperature of the source?

(c) Hydrogen freezes at 20.25 K and can be used to keep the D_2O moderator at a fixed temperature. What is the de Broglie wavelength λ_n (in Å) for neutrons with the average energy corresponding to that temperature? What is the average neutron energy E (in eV)?

(d) The neutrons are monochromatized at the 20.25 K thermal intensity maximum λ_n and diffracted off a crystalline target with lattice spacing d = 4.21 Å. What is the scattering angle θ that would give Bragg diffraction of the neutrons from this crystal? Use the convention for diffraction shown in the figure.



Problem 12

Three point–like particles of mass m are confined to move on a circle of radius r. Their mutual distances are fixed and equal (i.e. they lie on the corners of an equilateral triangle). Determine the rotational energy levels of the system, assuming the three particles have spin 0, i.e. they obey Bose statistics.