Physics Qualifying Examination

Problems 1–6	Thursday, January 8, 2009	1–5 pm
Problems 7-12	Friday, January 9, 2009	1-5 pm

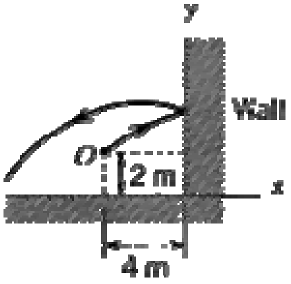
1. Solve each problem.

2. Start each problem solution on a fresh page. You may use multiple pages per problem.

3. At the top of each solution page put the problem number (1–12) and your Social Security number, but not your name or any other information.

Problem 1

A ball is thrown against a wall, making a perfectly elastic collision, and bounces over the head of the thrower as shown in the figure. When it leaves the thrower's hand, the ball is 2 m above the ground and 4 m from the wall, and has $V_x(0)=V_y(0)=10$ m/sec. How far behind the thrower does the ball hit the ground? (Numerical answer required, let $g=10$ m/sec^2.)



Problem 2

A pion of mass m_π decays into an electron and an antineutrino with masses m_e, and m_ν. Find the velocity of the antineutrino in the rest frame of the electron in terms of the masses of the three particles. Interpret your answer in the case where $m_\nu = 0$.

Problem 3

A particle of mass m and energy E approaches a one-dimensional step potential of height V_0, i.e. the potential is zero for $x<0$ and $V_0>0$ for $x>0$. Assume the particle is incident from the left.

(a) For $E<V_0$ determine the penetration depth, which is defined as the distance from $x=0$ where the probability to find the particle has dropped to $1/e$ of its value at $x=0$.

(b) For $E>V_0$ determine the reflection probability.

(c) For $E>V_0$ determine the transmission probability.

(d) For the case when E is much larger than V_0, find the leading approximation to the way in which the reflection probability varies with increasing energy.

(e) Suppose now that $V_0<0$, so that the step potential is a step downward, and $E>>|V_0|$. Is the reflection coefficient nonzero in this case? Explain! Is there a reflection classically?

Problem 4

Consider a *2*-level system with energy states ε and $\varepsilon + \Delta$ $(\Delta \geq 0)$.
(a) Compute the partition function and the free energy.
(b) Derive an expression for the specific heat $C(T)$.
(c) What are the low-T and high-T asymptotic forms of the above expression? Sketch your result.

Problem 5

Consider a spinless non-relativistic quantum particle of charge q and mass m moving in 3D in a homogeneous magnetic field $\vec{B} = B\hat{z}$ and subject to a parabolic potential $V(x, y, z) = \frac{1}{2}\kappa x^2$.

(a) Find a vector potential $\vec{A}$ in a gauge where $\vec{A}$ points in the $\hat{y}$ direction
(b) Write down the Hamiltonian operator for the above problem.
(c) Find the energy spectrum.

Problem 6

Consider a quantum mechanical system with three possible orthonormal states ("colors" red, blue and green). Any wave function $|\psi\rangle$ may be written as a linear combination of the three basis states $|R\rangle$, $|B\rangle$, and $|G\rangle$. The system is described by the Hamiltonian

$$H = E_0 \, (2|R\rangle\langle R| + 2|B\rangle\langle B| + 2|G\rangle\langle G| - |G\rangle\langle B| - |B\rangle\langle G|)$$

1. Use the vector basis $|R\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |B\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, |G\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, express the Hamiltonian as a matrix, and find the normalized energy eigenstates and their corresponding eigenvalues.

2. Assume that at time $t = 0$ the wave function is $|\psi(0)\rangle = |G\rangle$. Find the wave function $|\psi(t)\rangle$ at an arbitrary time *t*. What are the respective probabilities for the color to be measured as red, green, or blue at time t ?

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Problem 7

For each of the following energy quantities, give an expression in terms of fundamental constants which is dimensionally correct (numerical prefactors are not required) and compute from your expression an order of magnitude estimate in units of electron Volt (eV).

Useful information (formulas in Gaussian cgs units):
- Rydberg: $1\,\mathrm{Ry} = m_e e^4 / (2\hbar^2) \approx 13.6\ \mathrm{eV}$.
- Bohr Magneton: $\mu_B = e\hbar / (2m_e c) \approx 6\times 10^{-5}\ \mathrm{eV/T}\ (\mathrm{T\ Tesla})$.
- Fine Structure Constant: $\alpha = e^2 / (\hbar c) \approx 1/137$.

Example: The electron kinetic energy in the ground state of the Hydrogen atom.

Solution: Denote the size of the atom by a_B . From Newton's second law one finds $T = m_e v^2 / 2 = e^2 / (2a_B)$ for the kinetic energy and the quantization condition $m_e v a_B = \hbar$ leads to $T \sim \hbar^2 / (m_e a_B^2)$. When equated with the potential energy e^2 / a_B , this yields $a_B = \hbar^2 / (m_e e^2) \approx 5\times 10^{-11}\mathrm{m}$. The kinetic energy is then $T \sim m_e e^4 / \hbar^2 = 2\ \mathrm{Ry} \approx 10\ \mathrm{eV}$. (One Ry is 13.6 eV, but the order of magnitude 10 eV is sufficient for this problem).

1. The ground state binding energy of the electron in a U^{91-} ion (a Uranium ion with atomic number 92, atomic weight 238, which is 91–times ionized so that there is only one electron present).

2. The ground state energy level splitting in a Hydrogen atom due to the spin–orbit interaction (fine structure splitting).

3. The hyperfine splitting in a Hydrogen atom (the splitting of electronic energy levels due to the interaction with the magnetic moment of the proton).

4. The relativistic correction to the binding energy of a Hydrogen atom.

5. The Zeeman splitting of electron levels in a Hydrogen atom when the magnetic field is 1 T (T Tesla).

6. The energy of a nucleon in a typical nucleus.

7. The rotational energy of an H_2 molecule.

Problem 8

(a) Consider a photon gas at temperature T = 2.9 K. Using E = uV and dE = –PdV for an adiabatic, quasistatic process, derive an expression for how the energy density u depends upon the volume V for an adiabatic, quasi-static expansion of the gas. Hint: P = u/3 for a photon gas.

(b) For a photon gas, u ~ T^4. Using this, derive an expression for how the volume V depends upon the temperature T for an adiabatic, quasistatic process.

(c) If the universe expanded quasi-statically and adiabatically from an initial state where the radiation temperature was T = 3000 K to the current state with a temperature of T = 2.9 K, what is the ratio of the current volume to the initial volume? Assume that the universe is composed of no matter.

Problem 9

Estimate each of the following, with an explanation for your reasoning:

(a) the average kinetic energy of a monatomic air molecule in this room.

(b) the molar heat capacity at constant volume of a rock at room temperature.

(c) the electrical resistance of a metal wire one meter long and 1 mm in radius. (Hint: $\rho \sim 10^{-8}\,\Omega m$).

(d) the number of atoms in 1 cm^2 on the surface of a solid metal.

(e) the number of atoms in 1 m^3 of gas at room temperature and atmospheric pressure.

Problem 10

A positive charge is uniformly distributed throughout a very long cylindrical volume of radius R. The charge per unit volume is ρ.

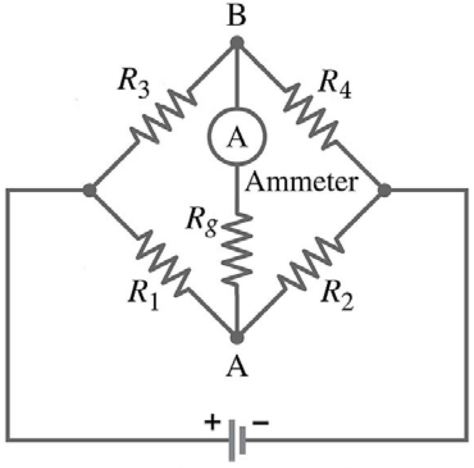
(a) Find the electric field $\bar{\boldsymbol{E}}$ everywhere as a function of the distance $\boldsymbol{r}$ from the axis of the cylinder.

(b) Find the electric potential V everywhere as a function of $\boldsymbol{r}$. Define $V = 0$ at the surface of the cylinder.

(c) Sketch $\boldsymbol{E}$ and V as function of $\boldsymbol{r}$, from $\boldsymbol{r} = 0$ to $\boldsymbol{r} = 3R$, showing the values of each at $\boldsymbol{r} = 0$, R, and $3R$.

Problem 11

Consider the Wheatstone bridge resistor circuit shown in the diagram below.



(a) Use Kirchhoff's laws to derive a condition for the values of R_1, R_2, R_3, and R_4, under which the galvanometer does not measure a current and the Wheatstone bridge is "balanced".

(b) What is the total resistance of the circuit? The total resistance is defined as the ratio of the battery voltage over the current through the galvanometer.

Problem 12

A very flexible uniform chain of length ***L*** and linear mass density σ is suspended from one end so that it hangs vertically, with the lower end just touching the surface of a scale. The upper end is suddenly released so that the chain falls onto the scale and coils up in a small heap, each link coming to rest the instant it strikes the scale. Calculate the force measured by the scale as a function of the distance ***x*** that the top of the chain has fallen in the following steps:

1) Find the velocity of the chain in terms of the distance ***x***.

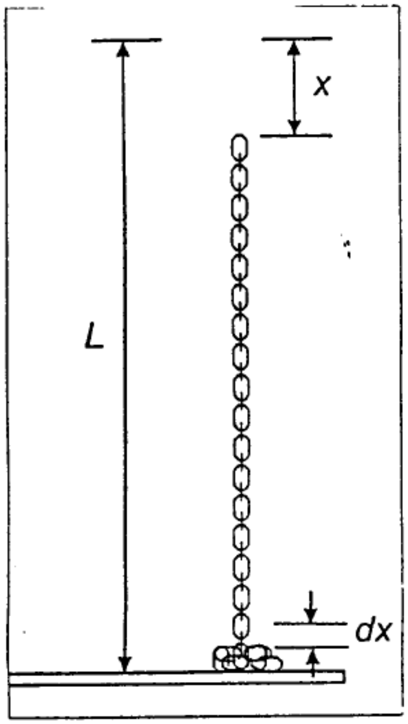
2) What is the force due to the momentum change as a function of ***x***?

3) What is the force due to the weight as a function of ***x***?

4) What is the total force in terms of ***x***?

5) What is the total force the instant before ***x*** = ***L***?

What is the final total force after the chain has completely fallen?



Chain, of total length L, has fallen a distance x. Length element dx (of mass dm) is about to be stopped by striking the table.