Appendix A

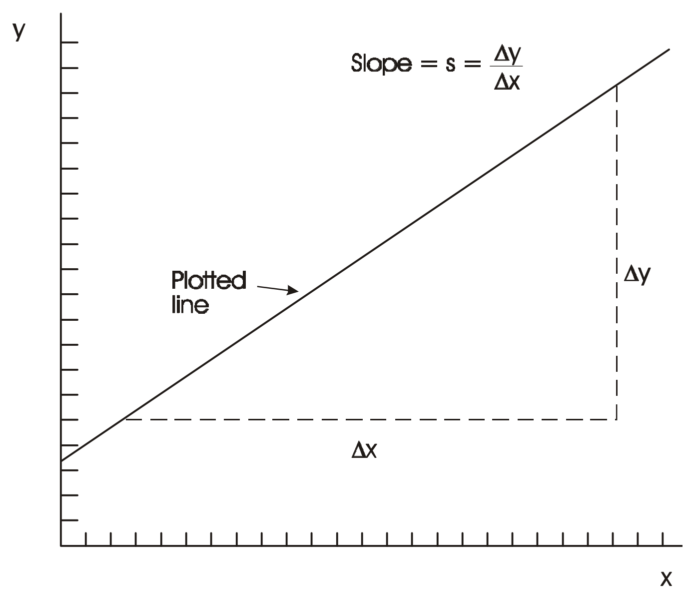
Graphing a Linear Function

Consider an object that is moving at constant velocity v_o, until at a certain time it is given a constant acceleration a in the direction of motion. In this case the velocity v at a later time is given by $v = v_o + at$ where t is the time measured from the time of application of a. (See Chapter 1 of your text). Knowing this fact, suppose you wanted to investigate experimentally to see if a certain object moving in a straight line was undergoing constant acceleration. Suppose, further, that to investigate this you made the following measurements of velocity at various times t.

V (m/s)	t (sec)
7.5	1.0
9.8	2.0
12.8	3.0
16.3	4.0
18.8	5.0
22.1	6.0
24.5	7.0
28.2	8.0

Suppose, also, that from studying your experimental arrangement you know that the time has been measured very accurately, but the measured velocities are uncertain to ± 0.5 cm/s.

How would you decide whether, within experimental error, your measurements indicate a constant acceleration? Probably the best way would be by plotting a graph of v versus t. While many people use computers now to make and analyze such plots, they do so only after they know how to make and interpret such plots manually. The purpose of this appendix is to review your knowledge of how to plot linear functions such as $v = v_o + at$. You may have studied linear functions is the standard form $y = mx + b$. For example, you may have plotted $y = 5x + 6$ and you may have determined that such a function yields a straight line or linear plot with slope $m = 5$, y intercept $b = 6$ and x intercept $-b/m = -6/5$. In any case, plot $y = 5x + 6$ for $-1 \le x \le 5$ on linear (ordinary) graph paper and determine *from your graph* the



slope and x and y intercepts. Remember that the slope is determined by using a section of your plotted line as the hypotenuse of a right triangle constructed as shown below and then taking the ratio of the change in y to the change in x. The y intercept occurs where x = 0 and the x intercept occurs where y = 0.

A *common mistake* made in taking the slope $\Delta y/\Delta x$ from *a graph on linear paper* is to take the ratio of the lengths of Δy and Δx. This works only if the same scales are used on the x and y axis and this is often not the case. In general, the changes Δx and Δy must be determined using the *x and y scales*.

Once you have made the above plot and determined the slope and intercepts, go back to the data table and plot v versus v_o. Assume that the data are consistent with the relationship $v = v_o + at$ and *determine a and v_o from your plot*. Since the data contain experimental error you will have to draw a line that, in your estimation, best represents the data. Your objective should be to minimize the deviation of the data points from the line. You should have nearly equal numbers of data points more or less randomly distributed on either side of the line. You should not require that your line necessarily pass through the beginning and end points as these are no more accurate than any others.

Appendix B

Log-Log Plotting

I. *Log-Log plot on a log paper*

It is often useful to plot the logarithm of one variable versus the logarithm of another when analyzing data. Such plots allow one to infer whether the relationship between the two variables is of the form $y = ax^n$ within experimental error. For if $y = ax^n$ then $\log y = n \log x + \log a$ and a plot of log y versus log x is a straight line of slope n and intercept log a.

Log-Log or full logarithmic graph paper has both abscissa and ordinate scales laid out in proportion to the logarithms of the variables. Thus these scales permit plotting log x and log y directly. There is, of course, no point on the scales representing the zero of either variable. Log-Log paper is available with various numbers of cycles along the axes. If the range of the variables to be plotted spans a factor of 10 or less, i.e., such as 0.1 to 1 or 1 to 10 etc., then 1 X 1 cycle paper is appropriate. If the variables span a range approaching 100 to 1 then 2 cycle X 2 cycle paper should be used, etc. In general, one chooses the number of cycles so that the plots are reasonably spread out over the paper. Log-Log paper also is available with a different number of cycles along the two axes.

It must be emphasized that the exponent n of the power law given in the above equation is the actual slope of the straight line fit to the data. This is measured by choosing a right triangle with most of the fitted straight line as the hypotenuse and using a *linear scale*, such as a ruler, to determine the length of the opposite and the adjacent sides. The ratio of these measured lengths will then be the slope n. This is true whether or not the log-log paper has the same number of cycles along both axes since in general the paper is made so that the individual cycles have the same physical dimension along each axis.

One of the disadvantages of log-log paper is that you may find yourself with paper with the wrong number of cycles for the data in hand. Generally, when the paper has too many cycles the temptation is to use it anyway. The result is a tiny graph. It is difficult to see any scatter in the data plotted on such a graph and it is difficult to obtain an accurate slope. Such small graphs are to be avoided in this lab.

II. *Log-Log plotting on Linear Paper*

The use of log-log paper was especially convenient when logarithms had to be looked up in tables or found on slide rules. Now with the availability of cheap scientific calculators, it is also convenient to take the logarithms of your data and plot these on linear paper. This has the major advantage that you can choose your scales on the x and y axis to avoid tiny graphs. *Note that when finding the slope of a graph on linear paper, you need to determine the ratio of the "rise" to the "run" from the scales and not from* *lengths measured with a ruler* (unless the scales along the x and Y axes happen to be the same). The y intercept (where log x = 0) is again log a. However, whereas a can be read directly from log-log paper, on linear paper a is found by taking the inverse logarithm of the intercept value.

Exercise

Plot $y = 8x^{3/2}$ on both 3 cycle x 3 cycle log-log paper and on linear paper for $0.4 \le x \le 20$. Verify that the slope is 3/2 and that the intercept or y value when x = 1 (log x = 0) is 8 in both cases.

A Further Point

Suppose you had wanted to plot $y = 8x^{3/2}$ for $0.4 \le x \le 8$. In this case y ranges from 2.024 to 181.02. Could you have plotted this on 2 cycle paper? At first it might seem that 3 cycles would be needed to cover the range of y, but, in fact, maximum and minimum y differ by less than a factor of 100 so two cycle paper should do. Two cycle paper can be used, in fact, if, for example, the function 1/2. $y = 1/2\times 8x^{3/2}$ is plotted instead of the original one. In this case, log 1/2y = 3.2 log x + log (1/2×8). The variable along the x axis ranges from 0.4 to 8 as before, but along the y axis the range is now 1.012 to 90.51 or less than 2 full cycles. The resulting graph will still have a slope of 3/2, the intercept, of course, now will be 4 or one-half the original value. Such an approach in which the original function is multiplied by an appropriate factor can often be used to minimize the number of cycles needed for plotting and maximize the spread of the graph.