

Experiment X: Waves and Resonance

Goals

- Study standing waves on a string under tension
- Experimentally determine the relationship between velocity of wave propagation, tension in the string, and the linear density of the string

Introduction and Background

Waves: A wave is the propagation of vibration in a medium. The velocity at which a vibrational wave propagates through a medium depends not only on the properties of the medium, but also on the external conditions imposed on it. For example, the velocity, V , with which a transverse vibrational wave travels down a string or flexible wire would depend on the linear density (mass per unit length), μ , of the string, as well as the tension, T , in the string. The tension supplies the restoring force when the string is given a transverse displacement, and the linear density determines the inertia of the string and thus its speed of response to the tension. Therefore, we would expect that the velocity would increase with increasing tension and decrease with increasing linear density. Based on these arguments we can assume a power law relationship between V , T and μ of the form

$$V = C \frac{T^{\alpha}}{\mu^{\beta}} \quad (10-1)$$

where C is a constant and α and β are the exponents. We expect C , α , and β to be positive integers or simple fractions. As discussed in Experiment III (Centripetal Force), the standard way to determine the power law exponents is to create a log-log plot and the slope of that plot would yield the exponent.

Standing Wave: If you vibrate one end of a string and keep the other end fixed, a continuous wave will travel down the string to the fixed end and be reflected back. Hence on the string there will be waves traveling in both directions and they will interfere with each other. Usually this results in a jumble. But with just the right combinations of the vibration frequency and tension in the string, the two traveling waves will interfere in such a way that there will be a large amplitude *fixed* pattern on the string, as shown in Figure 10-1. These fixed patterns are called *standing waves*, because they do not appear to be traveling. Since they have very large amplitude and occur only at certain particular frequencies, they are called *resonances*. The points of destructive interference, where the string remains still, are called *nodes*; and the point of constructive interference, where the string oscillates with maximum amplitude, are called *antinodes*. The nodes (or antinodes) are separated by exactly one half of a wavelength, λ . Namely, the condition for a resonance to occur is

$$L = n \frac{\lambda}{2} \quad (10-2)$$

where L is the length of the string and n is the number of antinodes on the string, as shown in Figure 10-1. The standing wave with n antinodes is called the n th harmonic. Therefore, from the number of antinodes we can determine the wavelength of the standing wave formed, and its velocity can be calculated from

$$v = \lambda f \quad (10-3)$$

where f is the frequency of the vibration.

In the actual experiment, you will get a string with a particular linear density. You will vary the tension in the string to produce a series of standing waves with different wavelengths. This series of data of V versus T will then be used to produce a log-log plot and determine the exponent α . Then we will use data from different groups (each group will get a string of different μ) for the same tension to produce the V versus μ data and determine the exponent β .

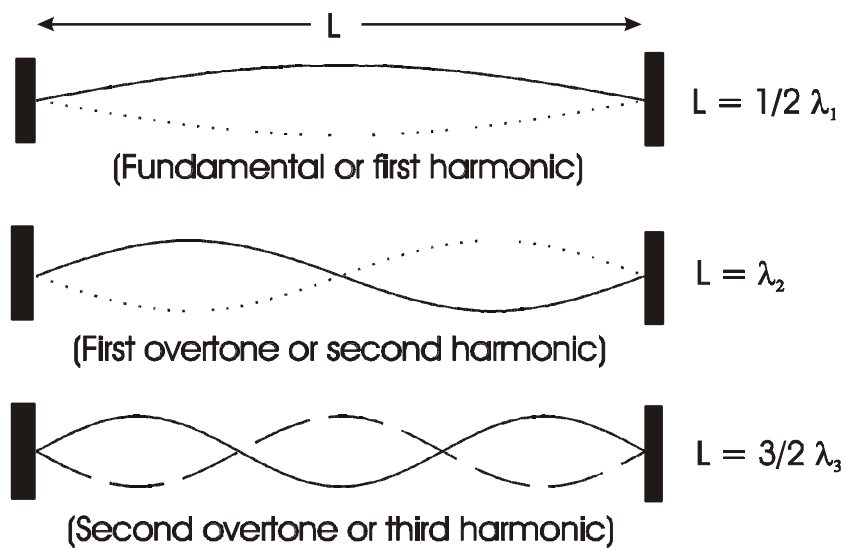


Figure 10.1 – Examples for first few harmonics.

Experimental Setup

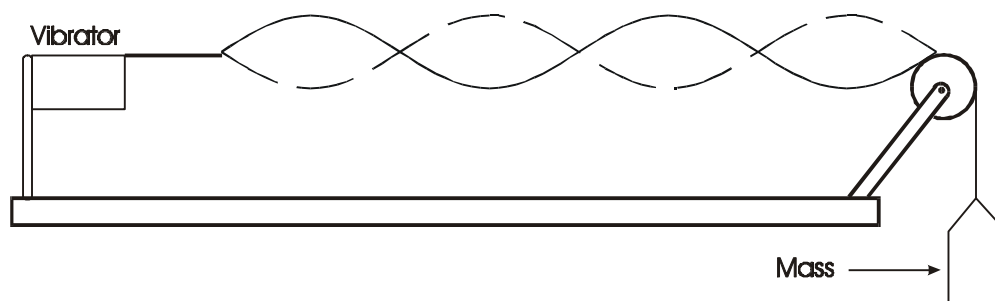


Figure 10.2 – Resonance on a vibrating string.

Equipment: 60 Hz electric vibrator, strings of different density, pulley, swivel clamp, table clamps, rods, weights, meter stick.

Setup: A schematic diagram of the experimental setup is shown in Figure 10-2. The string is held horizontally by the vibrator at one end and by weights hung over the pulley at the other end. The mass of the weights can be varied to produce different tension in the string.

Experimental Procedure and Data Analysis

1. Set up the apparatus as shown in Figure 10-2. There will be nylon strings of several different linear densities available. Your instructor will make sure that each group gets a string with different density.
2. Turn on the vibrator and adjust the tension to create standing wave patterns on the string. You should obtain at least 5 different standing wave patterns. It is a good idea to start with enough tension to get 3 or 4 antinodes. Then you can increase or decrease the tension to obtain other harmonics. Do not try to produce the first two harmonics as they require excessive tension.

The table below shows the approximate range of masses which must be hung on string of various strengths to produce the range of harmonics shown.

Line Strength	Range of Harmonics	Range of Hanging Mass (kg)
80 lb	4-9	2.0-0.40
60 lb	3-9	2.3-0.25
40 lb	3-9	1.1-0.10
20 lb	3-9	0.60-0.06
10 lb	3-7	0.30-0.05

It is useful to note the following:

- Because $V = \lambda f$, if you wish to decrease λ so as to produce more nodes, V has to be decreased by reducing the tension. Hence, the larger the tension, the fewer the nodes.
 - The vibrator is designed to vibrate at 60 Hz. However, certain combinations of vibrators, strings, and tensions may result in vibration at 120 Hz. Should you find that a particular tension produces a much shorter wavelength than expected, it is like that the vibration frequency is 120 Hz in that case. There is no need to discard these data. They are easily recognizable and you can fix them by using 120 Hz instead of 60 Hz to calculate V .
3. Record the distance between every other nodes, which should be the wavelength. Try to get as close to a resonance (maximum amplitude) as possible. Note that the string at the vibrator and the pulley are not quite “fixed” and are therefore not true nodes. Do not measure the wavelength from either of those points.

Calculate the wavelength and wave velocity on the string for each of the tensions. Input the values of λ , V , and T into the Excel template named “Waves”. Since the string density is constant (same string) in your experiment, from Equation 10-1 we have

$$\log V = \alpha \log T + \text{const.} \quad (10-4)$$

Calculate the values of $\log V$ and $\log T$, and make a linear plot of $\log V$ versus $\log T$. A straight line should result and the slope gives the exponent α .

To determine α , perform a Linear Regression Fit of $\log V$ versus $\log T$.

- What is the value of the slope?
 - What is the inferred theoretical value for α from the slope? Is it consistent with the expected value within experimental error?
4. To determine the exponent β , you need a set of data points (V , μ) at constant tension T . Since each group has used a string of different density, we can combine the data from different groups. Use the results of your linear regression (slope and intercept) from Part 3 to calculate the velocity V for a chosen common tension (e.g. 4.0 N).

Enter the combined values of V and μ into the second part of the template. Your lab instructor will give you the values of the linear density of the strings. Make a linear plot of $\log V$ versus $\log \mu$. Then perform a Linear Regression Fit of $\log V$ versus $\log \mu$.

- What is the value of the slope?
 - What is the inferred theoretical value for β from the slope? Is it consistent with the expected value within experimental error?
5. To determine the constant C in Equation 10-1, use the result of your calculation in Part 4. Enter the values of V , μ , T , α , and β for the common tension chosen in Part 4 into Equation 10-1, and calculate the constant C .
- What is the inferred theoretical value for C from the calculation?

Discussions and Questions

One simplest check for the correctness of an inferred relationship in physics is to see whether the relationship is dimensionally correct. Do your inferred values of α and β lead to a dimensionally correct result for V ?

Conclusions

Briefly discuss whether you have accomplished the goals listed at the beginning.