IM.3. The Geiger-Müller Counter

1. Purpose: Some measurements in nuclear decay, notions of statistics

2. Apparatus: Scaler-Timer (The Nucleus model 550),
Geiger-Müller tube,
oscilloscope,
radioactive sources.

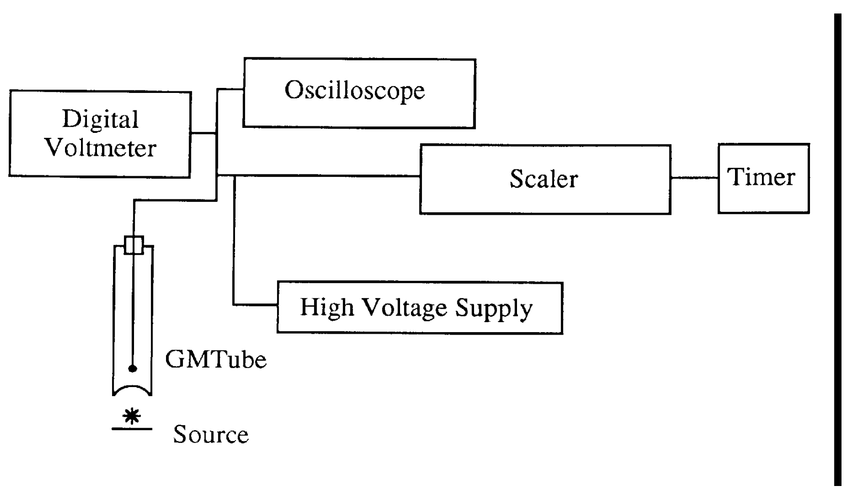
3. Introduction:

A typical Geiger-Müller (GM) Counter consists of a GM tube having a thin, mica end-window, a high voltage supply for the tube, a scaler to record the number of particles detected by the tube, and a timer which will stop the action of the scaler at the end of a preset interval.

The sensitivity of the GM tube is such that any particle capable of ionizing a single atom of the filling gas of the tube will initiate an avalanche of ionization in the tube. The collection of the ionization thus produced results in the formation of a pulse of voltage at the output of the tube. The amplitude of this pulse, on the order of a volt or so, is sufficient to operate the scaler circuit with little further amplification. However, the pulse amplitude is largely independent of the properties of the particle detected, and, therefore, can give little information as to the nature of the particle. In spite of this fact, the GM Counter is a versatile device in that it may be used for counting alpha particles, beta particles, and gamma rays with, however, varying degrees of efficiency.

4. Set-up:

Set up equipment as shown in Fig. 1. Scaler, Timer, and High Voltage Supply may well be contained in one package.



5. Measurements to be performed:
5.1 Characteristics of the GM counter:

Put a radioactive source below the GM tube. Put the counter in a counting mode and raise the voltage until counts are observed. Note the shape of the pulse and what happens as the voltage on the GM tube is increased. Do not exceed 900 volts at this time. What is the minimum voltage pulse necessary to activate the counter? After properly triggering the scope, sketch a picture of the pulse shape. How would you describe it?

Every GM tube has a characteristic response of counting rate versus voltage applied to the tube. A curve representing the variation of counting rate with voltage is called a plateau curve because of its appearance. The plateau curve of every tube that is to be used for the first time should be drawn in order to determine the optimum operating voltage. Find the plateau curve for your tube using the procedure outlined below.

A. Check to see that the high voltage as indicated by the meter on the instrument is at its minimum value.

B. Insert a radioactive source into one of the shelves of the counting chamber. Choose shelf and counting time such that you have at least 1000 counts in the plateau region.

C. Turn on the count switch and slowly increase the high voltage until counts just begin to be recorded by the scaler. The voltage at which counts just begin is called the "starting voltage" of the tube.

D. Beginning at the nearest 20 volt mark above the starting voltage, take one-minute (longer time if counting rate is too low) counts every 40 volts until a voltage is reached where a rapid increase in counts is observed. Reset scaler to zero before each count. Tabulate counts versus voltage.

E. Plot the data of (D). A plateau should be observed in the curve.

The optimum operating voltage will be about the middle of the plateau, usually some 150 to 200 volts above the knee of the curve. Set the high voltage to this point and record.

The slope S of the plateau of a GM tube serves as a figure of merit for the tube. The slope is defined to be the percent change in count rate per 100 volts change in applied voltage in the plateau region. A slope of greater than 10% indicates that the tube should no longer be used for accurate work.

The slope may be computed using

$$S\ (\%\text{ per }100\text{ V}) = \frac{2\,(R_2 - R_1)\,10^4}{(R_2 + R_1)\,(V_2 - V_1)} \qquad (1)$$

where V_2 is the voltage at the high end of plateau, R_2 is the count rate at this voltage, V_1 is the voltage at the low end of the plateau, and R_1 the corresponding rate. Do you understand this equation and can you explain it?

In order to compare, obtain a similar plateau curve for an old tube (if available).

5.2 Resolving time of the GM counter

There is an interval of time following the production of a pulse in the GM tube during which no other pulse can be recorded. This interval is called the resolving time of the system.
If this time is known it can be used to make a correction to the observed count rate to yield the true count rate. The procedure below can give a good estimate of the resolving time.

A. Obtain a resolving time source (a “split source”) from the instructor.
   This source is split into two parts. Remove one half of the source and set it aside.
B. Place the carrier containing one part on the second shelf of the counting chamber and make a trial count of 1 minute duration. Get the maximum count rate you can. Hopefully this should be more than 20,000 counts per minute, but if not use what you can get.
C. Make a 3-minute count and record the counts per minute, R_1.
D. Put the two parts of the source back together, taking care not to disturb the position of the first part. Make a 3-minute count of the combined parts and record as R_c.
E. Remove the part initially counted and make a 3 minute count on the second part. Record as R_2 (counts per minute).
F. Calculate the resolving time of the GM system using the relation:

$$\tau = \frac{R_1 + R_2 - R_c}{2R_1R_2} \qquad (2)$$

Convert the time thus found to microseconds/count and record. To understand the origin of the equation, see refs [1 - 3].

The resolving time τ may be used to correct an observed count rate using the expression:

$$R = r/(1-r\tau) \qquad (3)$$

where
r = Observed count rate
R = True count rate

To see the effect of the resolving time directly, use a very high rate source and view the pulses on the oscilloscope.

5.3 Statistical treatment of counting data

The emission of particles by radioactive nuclei is a completely random process. When, under identical conditions, a series of N measurements is made of the number of particles detected per unit time it will be observed that the individual measurements will vary about some average or mean value. The true mean, m, can be determined only by averaging an infinite number of measurements. However, for a finite number of observations the best approximation of the true mean is simply the arithmetic average $\bar{n}$

$$m \approx \bar{n} = (1/N)\,(n_1 + n_2 + n_3 + ... + n_N) \qquad (4)$$

The magnitude of the deviations of individual measurements from the true mean is usually expressed in terms of a Standard Deviation, σ. The Standard Deviation is defined to be the square root of the average value of the squares of the individual deviations (rms = "root-mean-square"). The number of counts of radioactive decays for a fixed time is a random variable whose probability distribution is a Poisson distribution; the Standard Deviation for such a distribution is simply the square root of the true mean:

$$\sigma = \pm\sqrt{m} \qquad (5)$$

The best approximation to the Standard Deviation of a finite series of measurements is given by the square root of the "sample variance",

$$\sigma_n = [\{(n_1 - \bar{n})^2 + (n_2 - \bar{n})^2 + ... + (n_N - \bar{n})^2\}/N\,]^{1/2} ... \qquad (6)$$

For values of $m > 20$ the Poisson distribution can be very well approximated by the Gaussian (or "normal") distribution for which certain confidence levels have been established in terms of the standard deviation. These confidence levels are as follows:

About 68% of the number of observations made will fall within the limits of $\bar{n} \pm \sigma_n$.
About 95% of the number of observations made will fall within the limits of $\bar{n} \pm 2\sigma_n$.
About 99% of the number of observations made will fall within the limits of $\bar{n} \pm 3\sigma_n$.

This means that if one additional measurement is made, it should have a 68% chance of falling within $\bar{n} \pm \sigma_n$.

When circumstances permit the making of only a single observation the number of counts obtained, n, is used as an estimator of and $\sqrt{n}$ as an estimator for its uncertainty (standard deviation σ_n).

The Standard Deviation of a gross counting rate, R_g is:

$$\sigma_{R_g} = \sqrt{n}\,/t = \sqrt{R_g t}\,/t = \sqrt{\frac{R_g}{t}} \qquad (7),$$

where t is the duration of the counting.

This expression is adequate when the sample counting rates is much higher then the background counting rate. However, when the background counting rate is appreciable compared to the sample counting rate the net counting rate, R, and its Standard Deviation, σ_r, are determined by

$$\sigma_R = [\sigma_{R_g}^2 + \sigma_{R_g}^2]^{1/2} = [\frac{R_g}{t_g} + \frac{R_b}{t_b}]^{1/2} \qquad (8)$$

where

R = Net counting rate,
R_g = Gross counting rate of sample, t_g = Period used to determine R_g,
R_b = Background counting rate, t_b = Period used to determine R_b.

To test the statistical nature of nuclear decay the following experiment can be performed:

A. Adjust the height of a source in the counting chamber to produce about 2000 counts per minute.
B. Take a set of 10 counts of 30 seconds duration.
C. Compute the arithmetic mean $\bar{n}$.
D. Compute the standard deviation σ_s of the mean (for a Poisson distribution of mean $\bar{n}$).
F. Compute the individual deviations from the mean (n_i - $\bar{n}$) and record in a table. Do they sum very nearly to zero?
G. Square the (n_i - $\bar{n}$), sum the square and apply Equation (6) to obtain the standard deviation σ_n. Compare σ_n with σ_s
H. Count the number of measurements whose values lie within $\bar{n} \pm \sigma_n$.

Now take a second set of ten measurements and repeat the same analysis. Compare the two mean values and sigmas. How many measurements of set 2 fall within the one sigma interval of the first set?

5.4 Background Measurements

Extraneous radiation called *background radiation* is always present. Gamma rays emitted by certain radioisotopes in the ground, the air, and various building materials as well as cosmic radiation from outer space can all provide counts in a detector in addition to those from a sample being measured. This background counting rate should always be subtracted from a sample counting rate in order to obtain the rate from the sample alone. Obtain a background counting rate using a 5-minute sample time.

5.5 Half-life determination of an unknown radioisotope

The activity (number of disintegrations per second) of a radioisotope is expressed as

$$A(t) = A_o e^{-\lambda t} \qquad (10)$$

where

A(t) = Activity at the end of an interval t
A_o = Activity at the beginning of the interval t
e = Base of natural logarithm
λ = Decay constant, characteristic of the radioisotope

The *half-life*, $T_{½}$ of a radioisotope is defined to be that interval during which the activity decreases to one-half its value at the beginning of the internal. In terms of half-life, the time dependence of the activity is

$$A(t) = A_o e^{-t \ln 2 / T_{½}} \text{ , where } T_{½} = \ln 2/\lambda \qquad (11)$$

The counting rate of a sample of a radioisotope may be considered to be directly proportional to the activity at the moment of measurement provided that the counting interval is short compared to the half-life. Reasonably short half-lives can be determined by measuring activity at regular intervals. The logarithm of the activity when plotted as a function of elapsed time should yield points falling in a straight line. Why?

Obtain your unknown sample from the instructor, measure the activity as a function of time and find the half-life.

6. Analysis, error estimation:

Resolving time:

Estimate the uncertainty on your resolving time measurement from the uncertainties on the number of counts (use Poisson uncertainties for these).

Half-life measurement:

You should take at least 20 measurements of the activity, for half-minute intervals. Having N measurements of activity gives you N-1 independent measurements of the half-life. Estimate the uncertainty on each individual half-life measurement from the uncertainties on the number of counts (remember that they are Poisson-distributed!). Determine the average of all of these values, and calculate the standard deviation. (If the uncertainties of these individual measurements are very different from each other, you may want to use a weighted rather than a straight average.)

Furthermore, you can also determine the half-life from the slope of the straight line in the plot of the logarithm of the activity versus elapsed time. The uncertainty on this slope can also be used to estimate an uncertainty on the half-life.

7. Report:

Your report should have a clear and complete discussion of the principles underlying the functioning of a GM Counter, as well as its characteristics as determined from your experimental data. In addition, you should have a complete description of the data analysis, including determination of uncertainties. You should treat every step in this experiment as a different measurement, with its data, analysis and conclusion together in one section.

7. References:

[1] Glenn F. Knoll: Radiation Detection and measurement,
John Wiley & Sons, New York 1989 (2nd ed.), 1999 (3rd ed.)

[2] William Leo: Techniques for Nuclear and Particle Physics Experiments :
A How-To Approach;
Springer Verlag, New York 1994 (2nd ed.)

[3] A. Melissinos: Experiments in Modern Physics, Academic Press, New York 1966

[4] http://polaris.phys.ualberta.ca/info/Phys29x/Manual/11GM01.pdf

[5] http://www.mathematik.uni-marburg.de/~kronjaeg/hv/radio/geiger/caltech/exp2.htm

[6] http://www.warren-wilson.edu/~sleavitt/formal/geigerwebpage.htm

[7] http://www.home.fh-karlsruhe.de/~mero0001/master/gm_counter.pdf

[8] http://www.home.fh-karlsruhe.de/~mero0001/master/deadtime.pdf

[9] http://www.lbl.gov/abc/wallchart/chapters/12/2.html

[10] http://www.astro.psu.edu/users/niel/astro485/derivations/geiger1.pdf