Physics 3221
Intermediate Mechanics
Problem Set 3
Due: Friday, Jan 28, 2005

3.1 Problem 2-9, Marion & Thornton, Pg. 91.

3.2 Problem 2-37, Marion & Thornton, Pg. 96.

3.3 A metal block of mass m slides on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a resistive force that varies as the 3/2 power of the speed:

$$F_r = -mkv^{3/2}.$$

The initial conditions are $x(t=0) = 0$ and $v(t=0) = v_0$.

(a) Solve the equation of motion for this block to find $v(t)$.

(b) Integrate $v(t)$ to obtain $x(t)$.

(c) Using the fact that

$$\frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx},$$

obtain a first order differential equation for v as a function of x. Solve this equation to find $v(x)$ for this block.

(d) Show that as $t \to \infty$ the displacement of the block asymptotically approaches $2v_0^{1/2}/k$.

3.4 A gun at ground level is fired straight up. Assume the air drag on the bullet varies *quadratically* with height. If we take the positive y direction to be up, the resistive force is then $F_r = -mkv^2$ when the particle is moving up, and $F_r = +mkv^2$ when the particle is moving down.

(a) Show that if the initial speed of the bullet is v_0 then, for upward motion, the speed of the bullet varies with height according to

$$v^2 = (v_0^2 - v_t^2)e^{-2ky} + v_t^2 \quad (\textit{upward motion}),$$

where y is the displacement measured from the ground, and $v_t = (g/k)^{1/2}$ is the terminal velocity.

(b) Now show that as the bullet falls its velocity varies with height according to

$$v^2 = v_t^2 - v_t^2 e^{2ky} \quad (\textit{downward motion}),$$

where now y is the displacement measured from the highest point of the bullets trajectory.

(c) Using the results of Parts (a) and (b) show that when the bullet hits the ground on its return, its speed is

$$\frac{v_0 v_t}{(v_0^2 + v_t^2)^{1/2}}.$$