Physics 3221
Intermediate Mechanics
Problem Set 5
Due: Friday, Feb 11, 2005

5.1 Consider a damped harmonic oscillator with $m = 1$ kg, $k = 50$ N/m and $b = 10$ kg/s. If the initial conditions are $x(t = 0) = 0.1$ m and $\dot{x}(t = 0) = 0$, find the subsequent motion of the system.

5.2 Consider a damped harmonic oscillator for which the equation of motion is

$$m\ddot{x} = -kx - b\dot{x}.$$

(a) By taking the time derivative of the total mechanical energy, $E = T + U$, of the oscillator and using the equation of motion show that the rate of energy loss is

$$\frac{dE}{dt} = -b\dot{x}^2.$$

(b) Show that for the case of a *critically damped* oscillator ($\beta = \omega_0$), for which the initial conditions are $x(t = 0) = x_0$ and $\dot{x}(t = 0) = 0$, the rate at which the oscillator loses energy is

$$\frac{dE}{dt} = -2m\beta^5 x_0^2 t^2 e^{-2\beta t}$$

(Remember that the damping parameter β is defined as $\beta = b/2m$.)

(c) For the oscillator of Part (b), compute the total energy lost after an infinite amount of time,

$$\text{Energy Lost} = \int_0^\infty \frac{dE}{dt} dt,$$

and verify that it is equal to the total initial energy of the oscillator.

5.3 If the amplitude of an underdamped oscillator decreases to $1/e$ of its initial value after n complete cycles, show that the ratio of the period of oscillation, τ_1, to the period of the same oscillator with no damping, τ_0, is

$$\frac{\tau_1}{\tau_0} = \left(1 + \frac{1}{4\pi^2 n^2}\right)^{1/2} \simeq 1 + \frac{1}{8\pi^2 n^2}$$

where the approximation is valid in the limit of large n.

5.4 Problem 3-22, Marion & Thornton, Pg. 140.

5.5 Consider a damped harmonic oscillator with $m = 10$ kg, $k = 250$ N/m and $b = 60$ kg/s. This oscillator is subject to a driving force given by $F_0 \cos \omega t$, where $F_0 = 48$ N.

(a) What value of ω results in steady-state oscillations with maximum amplitude?

(b) What is the value of the maximum amplitude?

(c) When ω has the value found in Part (a), what is the phase angle δ?