

Physics 3221
Intermediate Mechanics
Problem Set 6

Due: Friday, Feb 18, 2005

6.1 If a series LCR circuit is connected across the terminals of an electric generator that produces a voltage $\mathcal{E}(t) = \mathcal{E}_0 \cos \omega t$, the flow of electrical charge q through the circuit is given by the following second-order differential equation:

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = \mathcal{E}_0 \cos \omega t,$$

where L is inductance, R is resistance, and C is capacitance.

- (a) Verify the correspondence shown in Table 3-1 between the parameters of a driven mechanical oscillator and the above driven electrical oscillator.
- (b) Calculate the Q of the electrical circuit in terms of L , R and C .
- (c) Show that in the case of weak damping, Q can be written as $Q = R_0/R$, where $R_0 = \sqrt{L/C}$.

6.2 Problem 3-18, Marion & Thornton, Pg. 140.

6.3 Problem 3-28, Marion & Thornton, Pg. 142.

6.4 Consider an undamped harmonic oscillator described by the equation of motion

$$m\ddot{x} + kx = F(t).$$

The driving force $F(t) = F_0 \sin(\omega t)$ is turned on at time $t = 0$.

- (a) Find the general solution of the equation of motion for times $t \geq 0$ by adding the complementary solution (i.e., the general solution to the homogeneous equation with $F(t) = 0$) to a particular solution. For the particular solution try $x_p(t) = D \sin(\omega t)$ and determine the constant D .
- (b) For the the initial conditions $x(t = 0) = 0$ and $\dot{x}(t = 0) = 0$, show that the displacement of the system for times $t \geq 0$ is given by

$$x(t) = \frac{F_0/m}{\omega_0^2 - \omega^2} \left(\sin \omega t - \frac{\omega}{\omega_0} \sin \omega_0 t \right),$$

where $\omega_0 = \sqrt{k/m}$.

- (c) Take the $\omega \rightarrow \omega_0$ limit of the result of Part (b) to show that when the oscillator is driven at its natural frequency the displacement as a function of time is given by

$$x(t) = \frac{F_0/m}{2\omega_0} \left(t \cos \omega_0 t - \frac{1}{\omega_0} \sin \omega_0 t \right),$$

and comment on the physical meaning of this result.