1. Assume that the two protons in the ^{4}He nucleus are 1.4×10^{-15} m apart. ($e = 1.6 \times 10^{-19}$ [redacted]C, $m_p = 1.67 \times 10^{-27}$ kg) (Remember that the magnitude of the charge on the proton is the same as that on the electron.) GIVE UNITS FOR ALL YOUR ANSWERS

a) 5 pts Calculate the magnitude of the electric force between the two protons.

$$F = \frac{ke^2}{r^2} = \frac{(9\times10^9\,Nm^2/C^2)(1.6\times10^{-19}\,C)^2}{(1.4\times10^{-15}\,m)^2} = 117\,N$$

b) 5 pts If the nuclear force holding the protons (and neutrons) together were suddenly "turned off," what would be the initial acceleration of the protons due just to the electric force? (Don't be surprised if it's very large!)

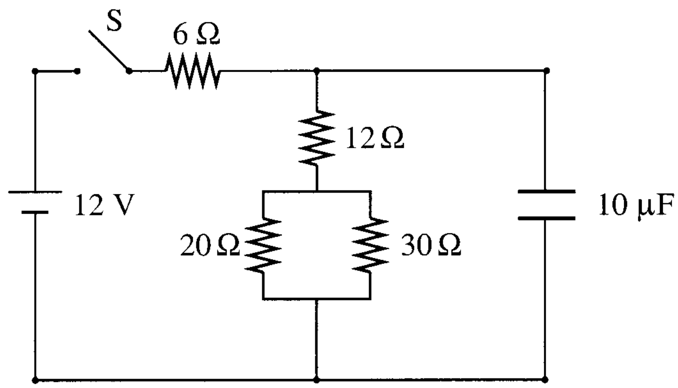
$$a = \frac{F}{m} = \frac{117\,N}{1.67\times10^{-27}\,kg} = 7.04\times10^{28}\,m/s^2$$

c) 5 pts Calculate the electric field of a bare ^{4}He nucleus (without any electrons) at a radius of 2.6×10^{-11} m. At this "large" radius (normally occupied by the electrons) we can ignore the separation between the two protons.

$$E = 2\frac{ke}{r^2} = \frac{(9\times10^9\,Nm^2/C^2)(1.6\times10^{-19}\,C)\times 2}{(2.6\times10^{-11}\,m)^2}$$

$$= 4.26\times10^{12}\,N/C$$

2. In the circuit shown, the batteries have negligible internal resistance. The capacitor is initially uncharged.



a) 5 pts Find the current through the battery immediately after the switch is closed.

$$I_i = \frac{12V}{6\Omega} = 2A$$

b) 5 pts Find the steady-state current through the battery long after the switch has been closed.

$$I_f = \frac{12V}{6\Omega + 12\Omega + \left(\frac{1}{20\Omega} + \frac{1}{30\Omega}\right)^{-1}} = 0.4A$$

c) 5 pts Find the charge on the capacitor long after the switch has been closed.

$$V = I_f \left(12\Omega + \left(\frac{1}{20\Omega} + \frac{1}{30\Omega}\right)^{-1}\right) = 9.6V$$

$$\Rightarrow Q = CV = (10\mu F)(9.6V) = 96\mu C$$

3. a) 5 pts Calculate the radius of the spiral path followed by electrons of kinetic energy 10 eV in the Earth's magnetic field of 6×10^{-5} T. ($m_e = 9.11 \times 10^{-31}$ kg)

$$v = \sqrt{\frac{2(10\,eV)(1.6\times10^{-19}\,J/eV)}{9.11\times10^{-31}\,kg}} = 1.87\times10^{6}\,m/s$$

$$\Rightarrow r_c = \frac{mv}{qB} = \frac{(9.11\times10^{-31}\,kg)(1.87\times10^{6}\,m/s)}{(1.6\times10^{-19}\,C)(6\times10^{-5}\,T)} = 0.178\,m$$

b) 5 pts Calculate the frequency of the spiral rotations under these conditions.

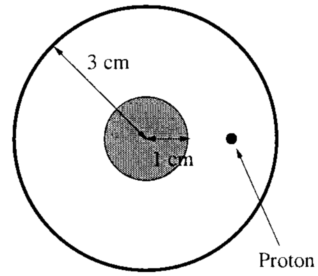
$$f_c = \frac{qB}{2\pi m} = \frac{(1.6\times10^{-19}\,C)(6\times10^{-5}\,T)}{2\pi(9.11\times10^{-31}\,kg)} = 1.68\times10^{6}\,Hz$$

c) 5 pts Calculate the frequency of the spiral rotations for electrons of kinetic energy 5 eV.

f_c is independent of v

$$\Rightarrow f_c = 1.68\times10^{6}\,Hz$$

4. An infinitely long solid wire of radius 1 cm carries a current of 20 A into the paper. Another infinitely long thin hollow cylinder 3 cm in radius carries a current of 20 A out of the paper.



a) 5 pts Calculate the magnetic field at a radius of 2 cm from the axis (between the conductors).

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\ Tm/A)(20\ A)}{2\pi\ (0.02\ m)} = 2 \times 10^{-4}\ T$$

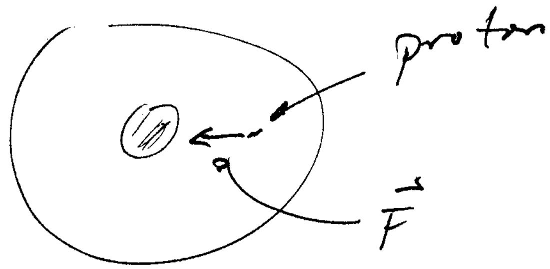
b) 5 pts Calculate the magnetic field at a radius of 5 cm (outside both conductors).

$$B = 0$$

c) 5 pts Calculate the magnetic force on a proton traveling into the paper at 10^6 m/s parallel to and at a distance of 2 cm from the axis (between the conductors). What is the direction of the force?

$$F = qvB = (1.6 \times 10^{-19})(10^6\ m/s)(2 \times 10^{-4}\ T)$$

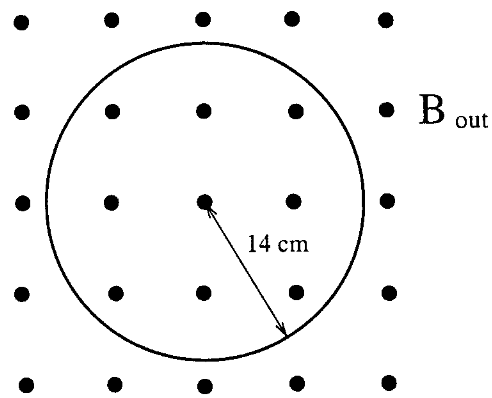
$$= 3.2 \times 10^{-17}\ N$$



5. A uniform magnetic field is normal to the plane of a circular wire loop of radius 14 cm as shown in the figure. The magnetic field points out of the page and increases as a function of time according to the relation

$$B = 0.06\, t^2 + 0.07$$

where B is in Tesla and t is in seconds. The loop has resistance 2 Ω.



a) 5 pts What is the value of the emf induced in the loop when $t = 2.0$ s?

$$\phi_m = \pi (0.14\,m)^2 (0.06\,t^2 + 0.07)$$

$$\frac{d\phi_m}{dt} = \pi (0.14\,m)^2 (0.12\,t) \underset{t=2s}{=} 1.48 \times 10^{-2}\ V$$

$$|\mathcal{E}| = 1.48 \times 10^{-2}\ V$$

b) 5 pts What is the magnitude of the current flowing around the loop at $t = 2.0$ s?

$$I = \frac{\mathcal{E}}{R} = \frac{1.48 \times 10^{-2}\ V}{2\,\Omega} = 7.39 \times 10^{-3}\ A$$

c) 5 pts Is the current flowing clockwise or counterclockwise around the loop as shown in the figure?

Clockwise by Lenz's law.

6. a) 5 pts A sinusoidal voltage source of amplitude 100 V and frequency 400 Hz is connected to a 25 Ohm resistor. Calculate the average power dissipated in the resistor.

$$I_{rms} = \frac{\mathcal{E}_{rms}}{R} = \frac{100V/\sqrt{2}}{25\Omega} = 2.83\ A$$

$$\Rightarrow P_{ave} = I_{rms}^2 R = (2.83A)^2(25\Omega) = \underline{\underline{200\ W}}$$

b) 5 pts A 3μF capacitor is now placed in series with the 25 Ohm resistor. Calculate the average power dissipated in the resistor in this circuit.

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi(400Hz)(3\times10^{-6}F)} = 132.6\ \Omega$$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{(25\Omega)^2 + (132.6\Omega)^2} = 135\ \Omega$$

$$\Rightarrow I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{100V/\sqrt{2}}{135\Omega} = 0.524A \Rightarrow P_{ave} = I_{rms}^2 R$$

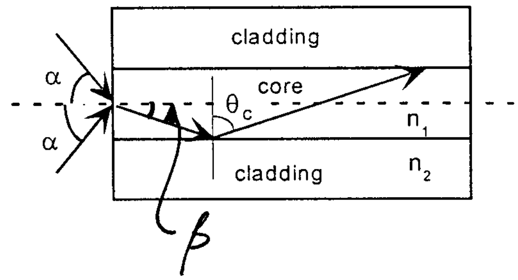
$$= (0.524A)^2(25\Omega)$$

$$= \boxed{6.86\ W}$$

c) 5 pts Calculate the average power dissipated in the capacitor.

$$\underline{\underline{P_{ave} = 0}}$$

7. (15 points) One type of optical fiber is composed of a central core, with refractive index n_1, surrounded by concentric cladding with lower refractive index, $n_2 < n_1$. The cross section of fiber is shown in the right figure.



(a) 5 pts What is the critical angle (θ_c) for total internal reflection in the core in terms of n_1 and n_2?

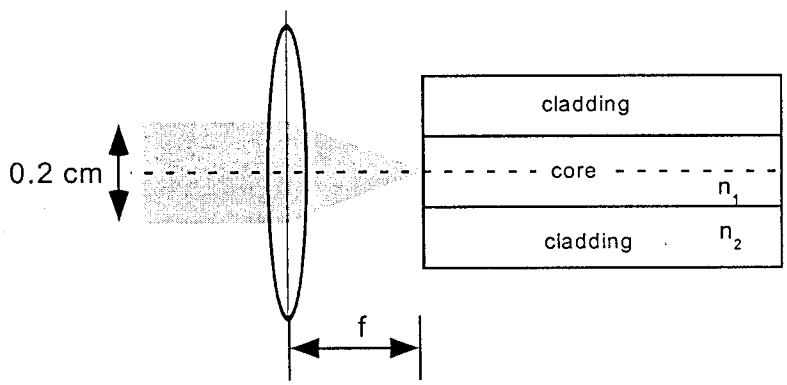
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

(b) 5 pts Given n_1=1.5 and n_2=1.45, what is the largest angle of incidence (α) that an incident ray from air (index of refraction = 1) can have for total internal reflection in the core?

$$\sin\alpha = n_1 \sin\beta \quad , \beta = 90° - \theta_c \Rightarrow \alpha = \sin^{-1}(n_1 \cos\theta_c)$$

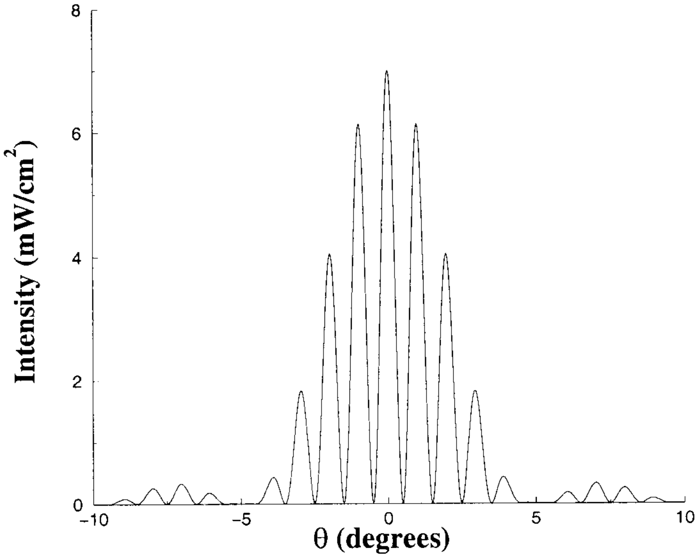
$$\Rightarrow \alpha = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right) = 22.6°$$

(c) 5 pts Image you are coupling a parallel laser ray with diameter of 0.2 cm into the core of the fiber with a converging lens, as shown in the figure below. What is the minimum focal length of the coupling lens you must choose in order to have no light leaking from core to the cladding? (n_1=1.5 and n_2=1.45)



$$\tan\alpha = \frac{0.1\,cm}{f} \Rightarrow f = \frac{0.1\,cm}{\tan 22.6°} = 0.24\ cm$$

8. Light of wavelength 440 nm passes through a double slit yielding the diffraction pattern of intensity I versus deflection angle θ shown in the figure.



a) 5 pts Find the width of each slit.

$a \sin\theta = \lambda \Rightarrow a = \frac{440\,nm}{\sin 5^\circ} = 5048\,\mathring{A}$

b) 5 pts Find the separation between the slits.

$d \sin\theta = 5\lambda \Rightarrow d = 5a = 25,240\,\mathring{A}$

c) 5 pts If the separation between the slits is doubled, but the width of the slits is unchanged, how many interference maxima will fall within the central diffraction maximum?

$9 + 9 + 1 = 19$