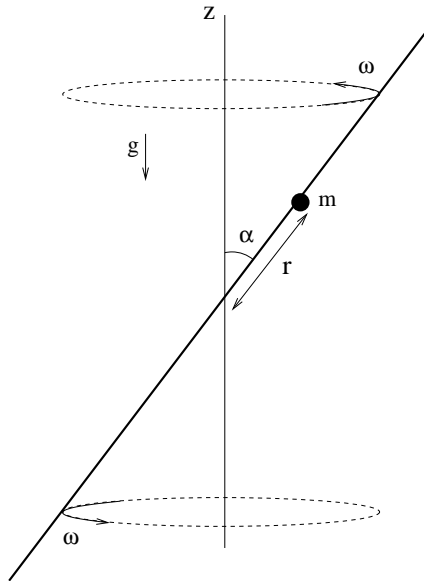


Theoretical Dynamics — PHY 5246

Midterm I October 22, 2003

1. A long straight wire makes an angle α with the z axis. The wire rotates about the z axis with a constant angular frequency ω as shown in the figure. Both α and ω are fixed. A bead of mass m slides on the wire without friction. Gravity acts.



- Write down the Lagrangian for this system in terms of the generalized coordinate r shown in the figure.
- Obtain the Euler-Lagrange equation for r .
- Find the energy function h for the Lagrangian found in Part (a). Is h conserved? Is it equal to the total energy? Explain your answers.
- Show that for a given ω there exists a solution with $\dot{r} = 0$. Find an expression for the value of r for which this occurs in terms of ω , g and α . Is this solution stable or unstable?

2. A particle of mass m moves in a central force field described by the potential function,

$$V(r) = kr, \quad k > 0.$$

- (a) Write the Lagrangian for this particle using the usual polar coordinates in the plane of the orbit, r and θ , as generalized coordinates.
- (b) Obtain the Euler-Lagrange equations for r and θ and combine them to obtain an effective one-dimensional problem for r .
- (c) Find the effective potential $V_{eff}(r)$ for this system. Sketch a plot of $V_{eff}(r)$ vs. r and discuss the possible orbits.
- (d) Consider a circular orbit of radius r_0 . Find an expression for the speed of the particle in this orbit in terms of k , m and r_0 .
- (e) Find the angular frequency of small radial oscillations about this circular orbit. Express your answer in terms of k , m and r_0 .