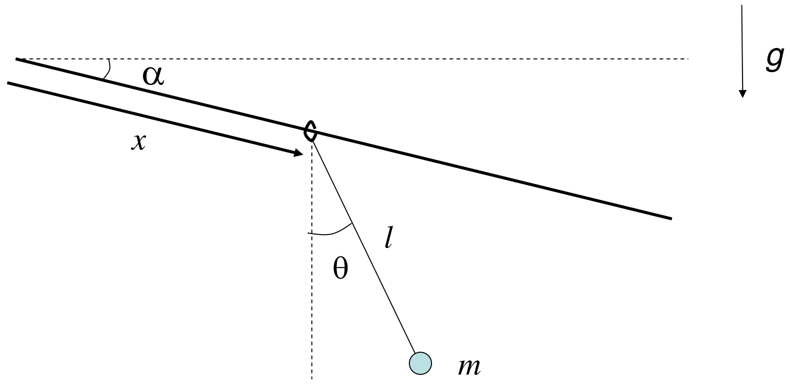
Theoretical Dynamics — PHY 5246

Midterm I   October 24, 2005

1. (30 pts) A pendulum consists of a mass m attached to the end of a massless rod of length l. The pivot point of the pendulum slides without friction on a straight wire which makes a fixed angle α with the horizontal (see figure). The pendulum is constrained to swing in the plane of the page.

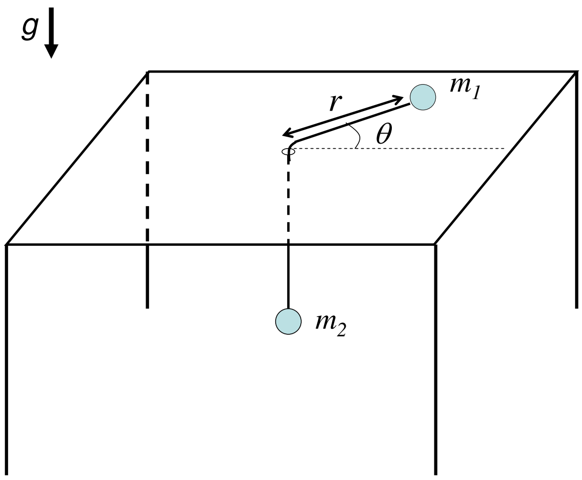


(a) Obtain the Lagrangian for this system in terms of the two generalized coordinates x and θ shown in the figure.

(b) Find the generalized momenta p_x and p_θ conjugate to x and θ. If $\alpha \neq 0$, are either of them conserved? What if $\alpha = 0$?

(c) For this system, is the energy function h equal to the total mechanical energy? Is it conserved? (You need not do any calculations for this part, but you should explain your answers.)

2. (50 pts) A mass m_1 slides without friction on the surface of a flat table. This mass is attached to a massless string which passes through a hole in the table and is connected to a second mass m_2 which hangs suspended and only moves in a vertical line (see figure).



(a) Write the Lagrangian for this system taking r and θ, as shown in the figure, to be the generalized coordinates.

(b) Obtain the Euler-Lagrange equations for r and θ and combine them to obtain an effective one-dimensional equation of motion for r.

(c) Obtain a first integral of the effective one-dimensional equation of motion for r found in (b) and interpret your result.

(d) Now consider a circular "orbit" for the mass m_1 of radius r_0. Find an expression for the speed of the mass m_1 in this orbit in terms of m_1, m_2, r_0 and g.

(e) Find the angular frequency of small oscillations about this circular orbit. Again, express your answer in terms of m_1, m_2, r_0 and g.

(f) Using the results of (d) and (e), compute the ratio of the period of the circular orbit to the period of the small oscillations.