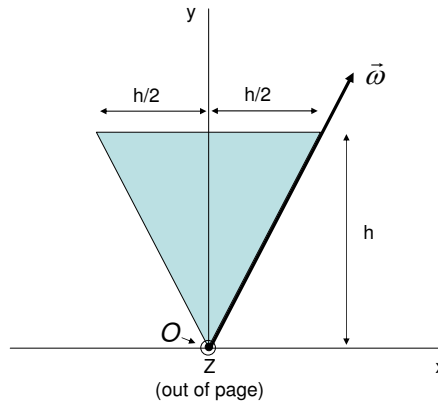


Theoretical Dynamics — PHY 5246

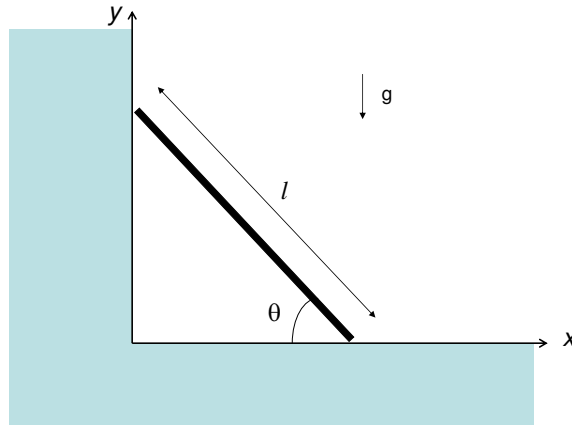
Midterm II November 22, 2006

1. (60 pts) Consider a uniform flat rigid body of mass M which has the shape of an isosceles triangle with height h equal to its base (see figure).



- (a) What are the principal axes of this body about the point \mathbf{O} ? (Give reasons for your answer based on symmetry.)
- (b) Determine the principal moments of inertia of this body about the point \mathbf{O} .
- (c) Assume this body is rotating with an angular velocity of magnitude ω about an axis that passes through the point \mathbf{O} and is parallel to one of the equal sides of the triangle (see figure). Find the angular momentum \vec{L} and the kinetic energy T of the body. Express your answer in terms of M , h , and ω .
- (d) Find the center of mass of this body.
- (e) Using the parallel axis theorem, determine the principal moments of inertia about the center of mass of this body.
- (f) About which principal axis or axes is torque-free steady rotation about the center of mass unstable for this body?

2. (40 pts) One end of a uniform rod of length l and mass m is on a vertical frictionless wall, and the other end is on a horizontal frictionless floor (see figure). The rod is confined to move in the xy plane and gravity acts. (Note: In this problem you need only consider the motion of the rod before it hits either the wall or the floor.)



- Express the kinetic energy of the rod in terms of the generalized coordinate θ shown in the figure and its time derivative. (You may use the fact that the moment of inertia of the rod about a perpendicular axis passing through its center of mass is $I = \frac{1}{12}ml^2$.)
- Find the potential energy of the rod in terms of the generalized coordinate θ and form the Lagrangian for this system.
- Find the Euler-Lagrange equation for θ .
- Assume that the rod is released from rest when $\theta = \theta_0$. Using conservation of energy, find an expression for the value of $\dot{\theta}$ just before the rod hits the floor.