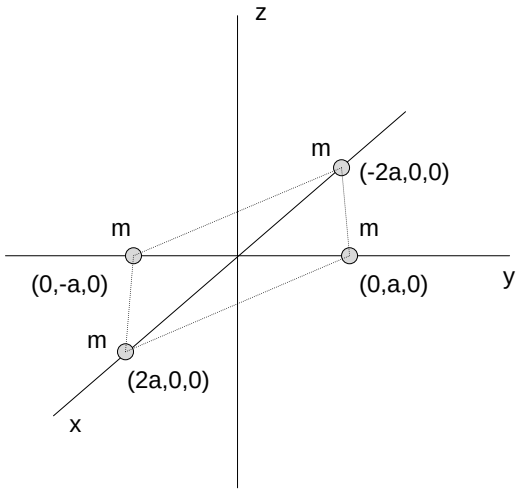
Theoretical Dynamics — PHY 5246

Midterm II November 24, 2003

1. A rigid body consists of four point masses, each of mass m, at the corners of a wire frame of negligible mass. The masses are located at the points $(x, y, z) = (2a, 0, 0)$, $(0, a, 0)$, $(-2a, 0, 0)$ and $(0, -a, 0)$ as shown in the figure below.



(a) Find the principal moments of inertia of the body about its center of mass. (Hint: You should be able to deduce the principal axes from symmetry.)

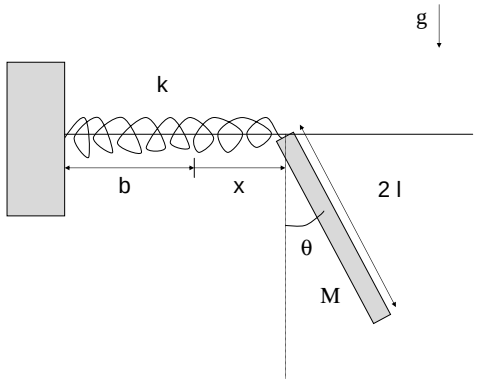
(b) If the body has an angular velocity about the center of mass of

$$\vec{\omega} = \omega(1, 1, 0)/\sqrt{2},$$

find the rotational kinetic energy of the body. What is the moment of inertia of the body about this rotation axis?

(c) Use the parallel-axis theorem to find the moment of inertia of the body about an axis parallel to the rotation axis of part (b) but passing through the point $(0, a, 0)$.

2. One end of a uniform bar of mass M and length $2l$ is confined to move along a horizontal axis. This end of the bar is also attached to a spring with spring constant k and equilibrium length b. The bar can swing freely only in the vertical plane. Let x be the displacement of the end of the spring from its equilibrium position and θ be the angle the bar makes with the vertical (see figure).



(a) Obtain the Lagrangian for this system in terms of the generalized coordinates x and θ.

(b) Obtain the **T** and **V** matrices suitable for studying small oscillations of this system.

(c) Find the eigenfrequencies for small oscillations of this system.