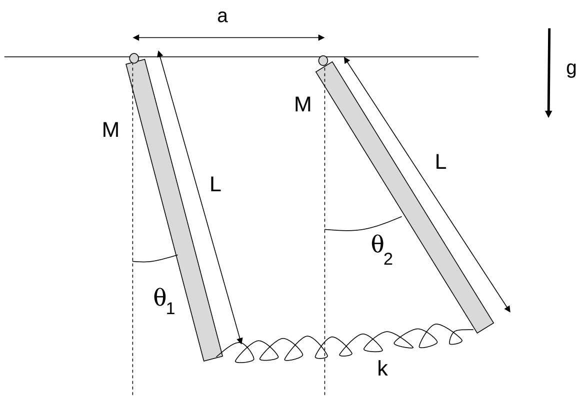
Theoretical Dynamics — PHY 5246

Midterm II November 19, 2004

1. (40 pts) A system of two coupled pendulums consists of identical uniform rods of length L and mass M connected by a spring of spring constant k as shown in the figure. The rods are constrained to move in the plane and gravity acts. The relaxed length of the spring is equal to a, the distance between the suspension points of the two rods, so the pendulums hang vertically when the system is in equilibrium.



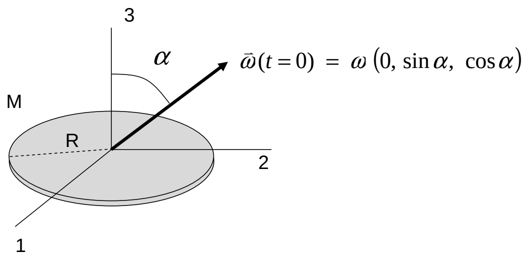
(a) Obtain expressions for the kinetic and potential energies of this system in terms of the generalized coordinates θ_1 and θ_2 and their time derivatives.

(b) Determine the **T** and **V** matrices appropriate for studying small oscillations of this system.

(c) Find the characteristic frequencies and corresponding eigenvectors for small oscillations of this system.

(d) Given the initial conditions at time $t = 0$ that $\theta_1(0) = \theta_2(0) = 0$ and $\dot{\theta}_1(0) = \Omega$, $\dot{\theta}_2(0) = 0$, find θ_1 and θ_2 for all subsequent times t.

2. (40 pts) A uniform solid disk of mass M, radius R, and negligible thickness undergoes torque free motion with its center of mass fixed. At time $t = 0$ the angular velocity of the disk in the body axes (labeled 123 in the figure) is $\vec{\omega}(0) = \omega(0, \sin\alpha, \cos\alpha)$, as shown below.



(a) Find the principal moments of inertia of this disk about its center of mass. (Even if you know the answer, please set up the relevant integrals.)

(b) Find the components in the body system of the angular momentum vector $\vec{L}$ of the disk about its center of mass at time $t = 0$. Sketch this vector in the figure above.

(c) Find the rotational kinetic energy of the disk. What is the moment of inertia of the disk about an axis parallel to $\vec{\omega}(0)$ passing through the center of mass?

(d) Solve Euler's equations for the motion of this disk and use the given initial condition at $t = 0$ to obtain the components of $\vec{\omega}$ in the body system for all subsequent times t.

Euler's equations for torque free motion:

$$
\begin{aligned}
I_1\dot{\omega}_1 &= \omega_2\omega_3(I_2 - I_3) \\
I_2\dot{\omega}_2 &= \omega_3\omega_1(I_3 - I_1) \\
I_3\dot{\omega}_3 &= \omega_1\omega_2(I_1 - I_2)
\end{aligned}
$$