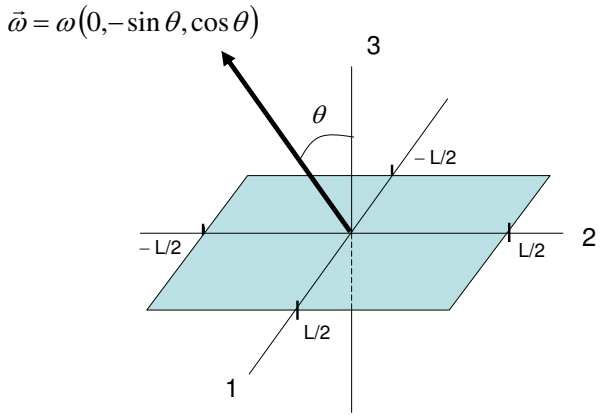
Theoretical Dynamics — PHY 5246

Midterm II November 30, 2005

1. (40 pts) Consider a thin uniform square tile of mass M, and side L. The tile is rotating with constant angular velocity $\vec{\omega}$ about a fixed axis passing through the center of the tile making an angle θ with the 3-axis (see figure).



(a) Find the principal moments of inertia of this tile about its center of mass.

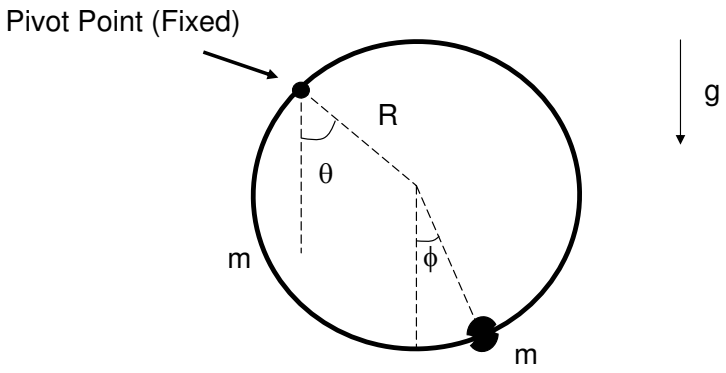
(b) Find the components in the body system of the angular momentum $\vec{L}$ of the tile about its center of mass.

(c) Find the components in the body system of the torque $\vec{N}$ about the center of mass that must be applied to the tile to maintain its constant angular velocity. **Hint:** Use the general result that for any time-dependent vector $\vec{G}$,

$$\left(\frac{d\vec{G}}{dt}\right)_{space} = \left(\frac{d\vec{G}}{dt}\right)_{body} + \vec{\omega} \times \vec{G}$$

to compute $\left(\frac{d\vec{L}}{dt}\right)_{space}$.

2. (40 pts) A uniform circular hoop of radius R and mass m hangs from a fixed pivot point and is free to swing in the plane of the paper. A bead, also of mass m, slides on the hoop without friction (see figure).



(a) Obtain expressions for the kinetic and potential energies of this system in terms of the generalized coordinates θ and ϕ and their time derivatives.

(b) Determine the $\mathbf{T}$ and $\mathbf{V}$ matrices appropriate for studying small oscillations of this system.

(c) Find the characteristic frequencies and corresponding eigenvectors for small oscillations of this system.

(d) Given the initial conditions at time $t = 0$ that $\theta(0) = 0$, $\phi(0) = \phi_0$, and $\dot{\theta}(0) = \dot{\phi}(0) = 0$, find θ and ϕ for all subsequent times t.