

PHY5246
Theoretical Dynamics
Problem Set 4

Due: Friday, Sept 29, 2006

4.1 Problem 12, GPS, Chapter 2, Pg. 65.

4.2 Problem 18, GPS, Chapter 2, Pg. 67. (**NOTE CORRECTED PROBLEM NUMBER!**)

4.3 Problem 20, GPS, Chapter 2, Pg. 67.

4.4 A particle of mass m is constrained to move under the influence of gravity on the surface of a paraboloid of revolution whose axis is vertical. Taking the z direction to be up, this surface is described by the equation

$$z = \alpha(x^2 + y^2); \quad \alpha > 0$$

- (a) Write the Lagrangian for this system using as generalized coordinates r and θ , the polar coordinates in the xy plane.
- (b) Reduce the problem to an effective one-dimensional problem for the radial coordinate r .
- (c) Determine the condition on the particle's initial velocity required to produce circular motion.
- (d) Find the period of small oscillations about this circular motion.

4.5 A particle of mass m moves in a central force field given by the potential

$$V = -k \frac{e^{-ar}}{r}; \quad k > 0, \quad a > 0.$$

- (a) Determine the effective one-dimensional problem for the radial coordinate. Sketch the effective potential for various values of l and discuss qualitatively the possible orbits of the system.
- (b) Determine the condition for a stable circular orbit. This condition should be expressed in terms of the radius of the orbit and the constant a .
- (c) Find the period of small radial oscillations about these circular orbits.