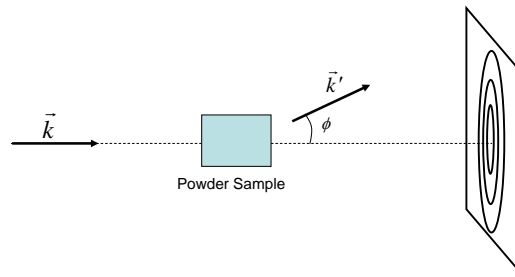


# Condensed Matter Physics I — PHZ 5491

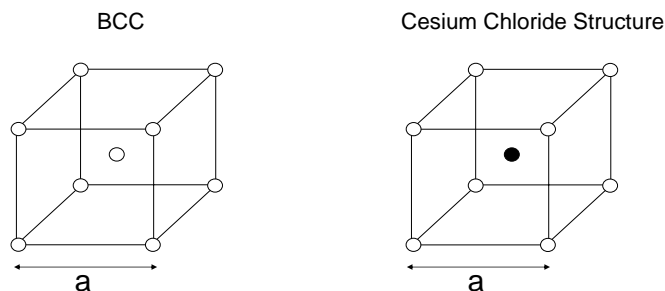
Midterm I    October 9, 2007

1. (50 pts) An x-ray diffraction experiment is carried out on a powder sample of a substance with BCC crystal structure. The smallest diffraction ring observed occurs at an angle of  $\phi = 50^\circ$ .



- (a) If the x-rays used in the experiment have a wavelength of  $1.0 \text{ \AA}$ , what is the lattice parameter  $a$  of the BCC structure?
- (b) For what angle  $\phi$  does the second smallest diffraction ring occur?

Now imagine the sample is replaced by a powder sample of a substance with *Cesium Chloride* crystal structure. The Cesium Chloride structure is similar to BCC except the atom in the center of the conventional unit cell is different from the atom on the corners (see figure). It should therefore be viewed as a simple cubic structure with a basis.



- (c) Assume the lattice parameter  $a$  of the new sample is the same as that of the original BCC sample. For what angle  $\phi$  is the smallest diffraction ring now observed?
- (d) Do you expect to observe more, fewer, or the same number of diffraction rings for the Cesium Chloride sample as for the BCC sample? Explain your answer.

2. (50 pts) Consider a gas of electrons in a one-dimensional solid with periodicity  $a$ . The electrons occupy an energy band with the following dispersion,

$$\mathcal{E}(k) = \mathcal{E}_0(1 - \cos ka), \quad -\frac{\pi}{a} \leq k < \frac{\pi}{a}.$$

(a) Show that the density of states  $g(\mathcal{E})$  (which in one dimension is defined to be the number of states per unit energy per unit length) for this energy band is

$$g(\mathcal{E}) = \begin{cases} \frac{2}{\pi a} \frac{1}{\sqrt{(\mathcal{E}_0)^2 - (\mathcal{E} - \mathcal{E}_0)^2}}, & 0 < \mathcal{E} < 2\mathcal{E}_0, \\ 0, & \text{otherwise.} \end{cases}$$

(b) Sketch  $g(\mathcal{E})$  vs.  $\mathcal{E}$  and indicate the van Hove singularities corresponding to the minimum and maximum of the band.

Now consider the case when this band is “half-filled” so that the Fermi energy is  $\mathcal{E}_F = \mathcal{E}_0$ .

(c) Use the Sommerfeld expansion (given below) to show that if  $\mathcal{E}_F = \mathcal{E}_0$ , then  $\mu = \mathcal{E}_F$  for  $k_B T \ll \mathcal{E}_0$ . (Hint: Use the fact that  $g'(\mathcal{E}_0) = 0$ .)

(d) Now use the Sommerfeld expansion to obtain an expression for the internal energy (per unit length)  $u(T)$  of the electrons, valid up to second order in  $k_B T / \mathcal{E}_0$ , when  $\mathcal{E}_F = \mathcal{E}_0$ . You may leave the temperature independent contribution to  $u(T)$  in the form of an integral. (Again, the fact that  $g'(\mathcal{E}_0) = 0$  should greatly simplify this calculation).

(e) Obtain an expression for the specific heat of the electrons as a function of temperature when  $\mathcal{E}_F = \mathcal{E}_0$  for  $k_B T \ll \mathcal{E}_0$ .

For this problem you need the first two terms of the Sommerfeld expansion,

$$\int_{-\infty}^{\infty} h(\mathcal{E}) f(\mathcal{E}) d\mathcal{E} = \int_{-\infty}^{\mu} h(\mathcal{E}) d\mathcal{E} + \frac{\pi^2}{6} (k_B T)^2 h'(\mu) + O((k_B T)^4),$$

where

$$f(\mathcal{E}) = \frac{1}{e^{(\mathcal{E} - \mu)/k_B T} + 1}$$

is the Fermi function.