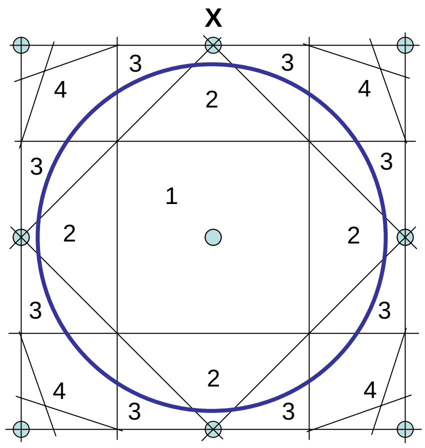
Condensed Matter Physics I — PHZ 5491

Midterm II November 15, 2007

1. (50 pts) Consider a two-dimensional metal consisting of a gas of electrons moving in a weak periodic potential due to a square lattice with lattice constant a. Assume the number density of electrons is $n = 5/a^2$, i.e. five electrons per primitive cell.

(a) Determine the Fermi wave vector k_F for free electrons in two dimensions with number density $n = 5/a^2$.

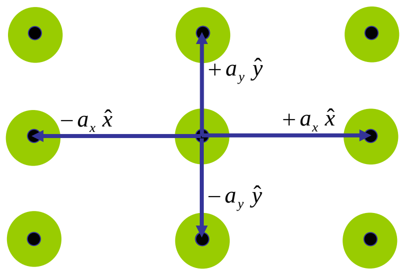
(b) From the result of (a) verify that the free electron Fermi surface is entirely contained within the first four Brillouin zones. To do this it is sufficient to show that k_F does not quite reach the point labeled **X** in the figure below. (In this figure the blue dots are reciprocal lattice points and numbers label the different Brillouin zones.)



(c) In the reduced zone scheme carefully sketch the nearly free electron Fermi surfaces in the first four zones. In each case indicate which states are occupied and which states are unoccupied.

(d) If a magnetic field is applied in the z direction (i.e. perpendicular to the two-dimensional gas) indicate for each of the zones in Part (c) whether the carriers contribute positively or negatively (or zero in the case of a filled band) to n_{eff}, the effective carrier density which determines the Hall coefficient. Do you expect the total n_{eff} to be positive or negative?

2. (50 pts) Consider a two-dimensional monatomic solid in which the atoms form a rectangular Bravais lattice with primitive vectors $\vec{a}_1 = a_x\hat{x}$ and $\vec{a}_2 = a_y\hat{y}$ where $a_x > a_y$ (see figure).



The tight-binding s-band for this lattice is determined by the matrix elements

$$\gamma(\vec{R}) = -\int d\mathbf{r}\phi_s(\vec{r})\Delta U(\vec{r})\phi_s(\vec{r}-\vec{R}),$$

where $\phi_s(\vec{r})$ is an atomic s-orbital associated with the atom at the origin and $\Delta U(\vec{r})$ is the perturbation due to the other ions in the lattice.

By symmetry we know that $\gamma(+a_x\hat{x}) = \gamma(-a_x\hat{x})$ and $\gamma(+a_y\hat{y}) = \gamma(-a_y\hat{y})$ and so we can define γ_x and γ_y as follows:

$$\gamma_x = \gamma(\pm a_x\hat{x}); \quad \gamma_y = \gamma(\pm a_y\hat{y}).$$

(a) Show that the tight-binding s-band for this system has the form

$$\mathcal{E}(\vec{k}) = \mathcal{E}_0 - 2\gamma_x \cos k_x a_x - 2\gamma_y \cos k_y a_y.$$

(b) Taylor expand $\mathcal{E}(\vec{k})$ about the point $\vec{k} = 0$ and show that near this point the dispersion is quadratic and can be expressed in the following form,

$$\mathcal{E}(\vec{k}) \simeq Const. + \frac{\hbar^2}{2}(k_x, k_y)\cdot\mathbf{M}^{-1}\cdot\begin{pmatrix} k_x \\ k_y \end{pmatrix}.$$

Determine the matrix elements of the (2×2) inverse effective mass tensor $\mathbf{M}^{-1}$ in terms of γ_x, γ_y, a_x, a_y and $\hbar$.

(c) Now imagine a magnetic field $\vec{B}$ is applied in the positive z direction (out of the page). Sketch a typical cyclotron orbit in k space near the point $\vec{k} = 0$ where the quadratic dispersion given above is valid. On this sketch clearly indicate the direction in which the orbit is traversed.

(d) Obtain the period of this cyclotron orbit as a function of γ_x, γ_y, a_x, a_y, $\hbar$, c, e and B.