

Physics 5690
Quantum Many-Body Physics
Problem Set 2

Due: Monday, Feb 10, 2003

2.1 Problem 2, Chapter 1 of N&O, Pg. 38.

2.2 Problem 4, Chapter 1 of N&O, Pg. 39.

2.3 Problem 5, Chapter 1 of N&O, Pg. 40.

2.4 Problem 4, Chapter 2 of N&O, Pg. 132.

2.5 Gaussian Integrals over Grassmann Variables.

(a) For the case $N = 2$, verify the following expression,

$$\int \prod_{\alpha=1}^N d\xi_{\alpha}^* d\xi_{\alpha} \exp \left(- \sum_{\alpha, \beta=1}^N \xi_{\alpha}^* M_{\alpha\beta} \xi_{\beta} + \sum_{\alpha=1}^B (\eta_{\alpha}^* \xi_{\alpha} + \xi_{\alpha}^* \eta_{\alpha}) \right) = \det M \exp \left(\sum_{\alpha, \beta=1}^N \eta_{\alpha}^* M_{\alpha\beta}^{-1} \eta_{\beta} \right),$$

where $\{\eta_{\alpha}, \eta_{\alpha}^*, \xi_{\alpha}, \xi_{\alpha}^*\}$ are Grassmann variables. Do this by Taylor expanding the exponentials and directly performing the Grassmann integrations.

(b) Again by direct Grassmann integration verify for the $N = 2$ case that

$$\int \prod_{\alpha=1}^N d\xi_{\alpha}^* d\xi_{\alpha} \xi_{\gamma}^* \xi_{\lambda} \exp \left(- \sum_{\alpha, \beta=1}^N \xi_{\alpha}^* M_{\alpha\beta} \xi_{\beta} \right) = (M^{-1})_{\gamma\lambda} \det M.$$