Physics 5690
Quantum Many-Body Physics
Problem Set 4
Due: Friday, Mar 21, 2003

4.1 **Labeled Diagrams and Symmetry Factors**.

(a) Enumerate the 24 terms which appear at 2nd order in the expansion of the grand partition function for the Hamiltonian

$$H = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha} + \frac{1}{2} \sum_{\alpha\beta\mu\nu} (\alpha\beta|V|\mu\nu) a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\nu} a_{\mu}.$$

Indicate the complete contraction and draw the labeled Feynman diagram for each term.

(b) Each of these diagrams is invariant under the group of $2!(2)^2 = 8$ operations formed by combining left-right interchanges at vertices and permuting time labels of vertices. The symmetry factor of a diagram is the number of these operations which leave that diagram invariant, i.e. which do not generate a different Wick's theorem contraction. For the 24 diagrams obtained in Part (a), compute the symmetry factor S. Then group the 24 labeled diagrams into sets of $2!(2)^2/S$ diagrams corresponding to a single *unlabeled* diagram with symmetry factor S.

(c) Evaluate each of the unlabeled diagrams for the case of spinless free particles (i.e. for the case where the noninteracting states are plane waves) interacting via a δ-function potential $V(r_i - r_j) = V_0 \delta(r_i - r_j)$. Show that in the fermion case the diagrams sum to zero when the symmetry factors are attached to each diagram. Why does this happen?

4.2 Problem 8, Chapter 2 of N&O, Pg. 134.

4.3 Problem 10, Chapter 2 of N&O, Pg. 134.