Intermediate Mechanics - PHY3221 Name:____________________________

Final Exam — Apr. 27, 2005

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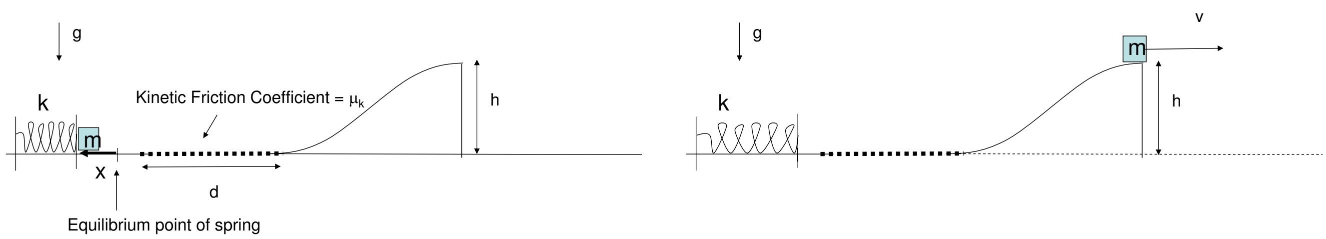
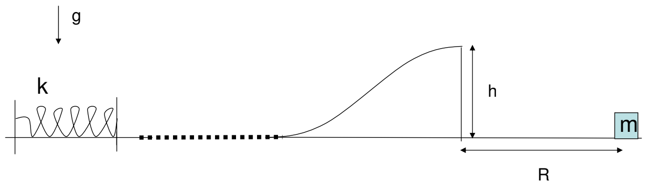
1. (10 pts) A block of mass m is launched using a spring of spring constant k which has been compressed from its equilibrium configuration by a displacement x (see figure). When the spring is released the block slides first on a level surface which is frictionless except for a rough patch of distance d with coefficient of kinetic friction μ_k. After the rough patch the surface is again frictionless and rises to a height h. (See figure.)



(a) Find an expression for the velocity of the block when it reaches the height h. Express your answer in terms of m, g, k, x, μ_k, d, and h.

When the block reaches the height h assume its velocity is purely in the horizontal direction. The block then flies off of a cliff and lands a distance R from the base of the cliff at its original height (see figure).



(b) Find an expression for the distance R from the base of the cliff at which the block hits the ground. Again, express your answer in terms of m, g, k, x, μ_k, d, and h.

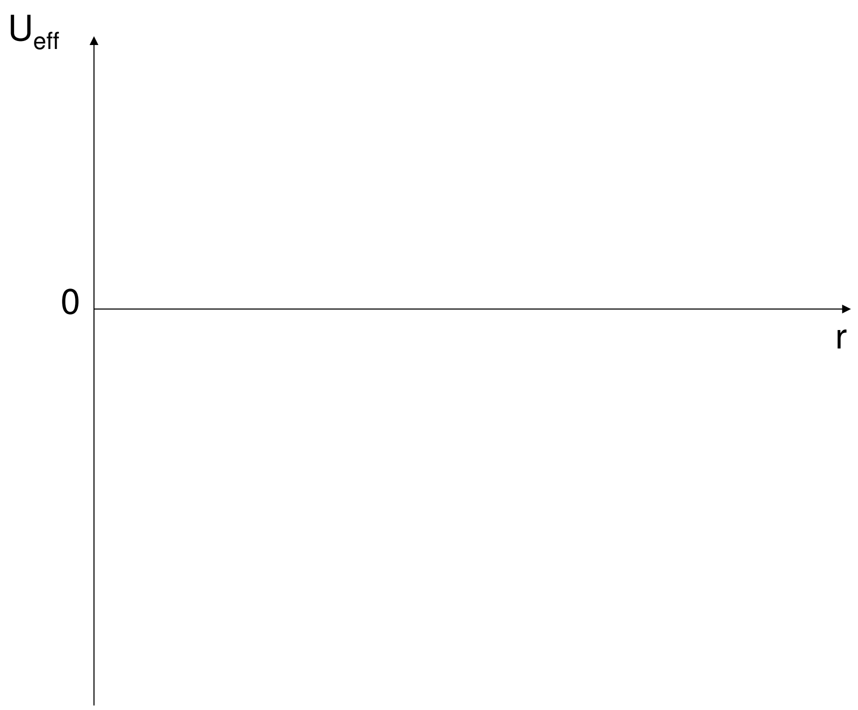
2. (10 pts) Consider a particle of mass m moving in the presence of a central force for which the potential energy function is

$$U(r) = -\frac{k}{r},$$

where k is a positive constant (i.e., the Kepler problem). The magnitude of the angular momentum of the particle about the center of force is l. (Here the center of force is fixed in space, so you may assume the mass m is the same as the reduced mass.)

(a) Write down an expression for the *effective* potential for this particle.

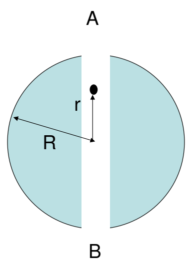
(b) Sketch a plot of the effective potential vs. r using the axes provided below. (Assume that l is not equal to zero). Indicate clearly on the y axis those energies E for which the orbit of the particle is a circle, an ellipse, a parabola, and a hyperbola.



(c) By minimizing the effective potential with respect to r, find an expression for the radius of a **circular** orbit for this particle. Your answer should be expressed entirely in terms of m, k and l.

(d) Find an expression for the distance of the particle from the center of force at perihelion (closest approach) for a **parabolic** orbit. Your answer should again be expressed entirely in terms of m, k and l.

3. (10 pts) A hole has been drilled straight through a spherical planet of uniform density with mass M and radius R. The hole passes through the center of the planet as shown in the figure. A stone of mass m is released from rest at the point A.



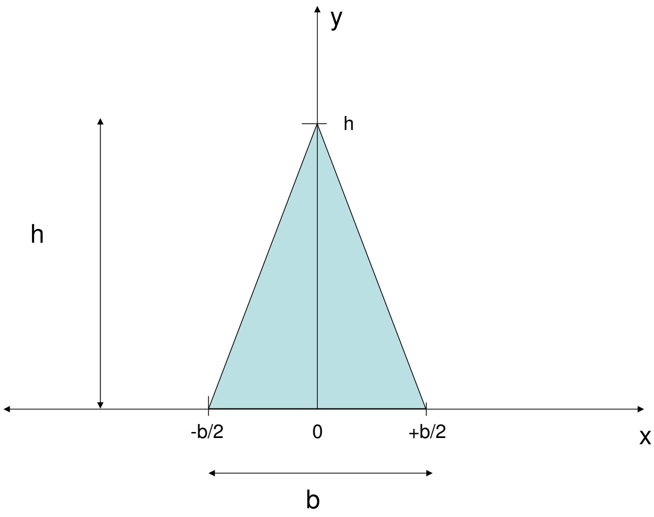
(a) What is the gravitational field due to the planet at a point in the hole which is a distance r from the center of the planet?

(b) Write down the equation of motion for the stone as it falls through the hole. What kind of motion does this equation describe?

(c) How long does it take the stone to go from point A to point B?

(d) Now imagine the stone is launched into a circular orbit just above the surface of the planet. For this orbit, how long does it take the stone to go from point A to point B? Compare your answer to that of Part (c).

4. (10 pts) Consider a solid isosceles triangle with uniform mass per unit area. The triangle has height h and base b, as shown in the figure.



Find the center of mass of this triangle.

5. (10 pts) A mass m_1 moving with initial velocity u collides with a stationary mass m_2. After the collision the two masses stick together and move with velocity v. The amount of energy dissipated during the collision is Q.

(a) Write down expressions for conservation of energy and momentum for this system.

(b) Find expressions for the initial and final velocities, u and v, in terms of Q, m_1 and m_2 only.