Statistical Mechanics — PHY 5524

Midterm February 27, 2008

1. (20 pts)

(a) Describe in a few sentences the conditions for which the classical partition function of an N particles system is given by $Q_N = (Q_1)^N$ and those for which it is given by $Q_N = \frac{1}{N!}(Q_1)^N$. (Here Q_1 is the partition function for a single particle.)

(b) Without performing any lengthy calculations, use the equipartition theorem to determine the specific heat of a system of N noninteracting three-dimensional classical harmonic oscillators.

2. (80 pts) Consider a classical system of N identical noninteracting diatomic molecules in a box of volume V at temperature T. The Hamiltonian for a single molecule is

$$H(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2) = \frac{1}{2m}(\vec{p}_1^{\,2} + \vec{p}_2^{\,2}) + K|\vec{r}_1 - \vec{r}_2|,$$

where $\vec{p}_1$, $\vec{p}_2$ and $\vec{r}_1$, $\vec{r}_2$ are the momenta and coordinates of the two atoms in the molecule.

(a) Show that the partition function for a single molecule is

$$Q_1(T, V) = V\left(\frac{2\pi m k_B T}{h^2}\right)^3 \left(8\pi\left(\frac{k_B T}{K}\right)^3\right).$$

(Hint: in doing the $\vec{r}_1$ and $\vec{r}_2$ integrations, consider changing variables to relative, $\vec{r} = \vec{r}_1 - \vec{r}_2$, and center of mass, $\vec{R} = (\vec{r}_1 + \vec{r}_2)/2$, coordinates.)

(b) Determine the partition function for N molecules, and obtain the Helmholtz free energy of the gas.

(c) Obtain expressions for the pressure and entropy of this gas as a function of T, V and N.

(d) Determine the internal energy and the specific heat at constant volume for this gas.

(e) Assume the gas initially has a volume V_0 and temperature T_0. If the gas expands adiabatically (i.e., with $\Delta S = 0$) to a final volume of $2V_0$, what is the final temperature of the gas?

Some useful integrals:

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}, \qquad \int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{\sqrt{\pi}}{4\alpha^{3/2}}$$