

**Physics 5524**  
**Statistical Mechanics**  
**Problem Set 4**  
Due: Wednesday, Feb. 6

4.1 (This is a modified version of a problem from the Fall 1997 Comprehensive Exam.)

Consider a classical one dimensional gas of impenetrable rods of mass  $m$  and length  $a$ . The rods cannot overlap, but otherwise do not interact. Let  $L$  denote the length of the system and  $N$  denote the number of rods.



- (a) Show that in the limit  $L \gg Na$  the partition function of this gas is approximately given by

$$Q_N(T, L) \simeq \frac{(L - Na)^N}{N!} \left( \frac{2\pi m k_B T}{h^2} \right)^{N/2}. \quad (1)$$

- (b) Assuming the above approximate expression for  $Q_N$  is valid, calculate the Helmholtz free energy of this gas as a function of  $T$  and  $L$ .
- (c) Find the equation of state for this gas relating the pressure  $P$ , the length  $L$ , and the temperature  $T$ . (In one dimension the pressure is simply the force the gas exerts on either end of the system).
- (d) Determine the entropy for this gas as a function of  $T$  and  $L$ .
- (e) How much work is done by this gas if it expands isothermally from size  $L_1 = 2Na$  to size  $L_2 = 3Na$ ?  
**(Note:** Strictly speaking in this part (and part (f)) we are not in the limit  $L \gg Na$ , nonetheless, make the assumption that the approximations leading to (1) are still valid.)
- (f) By what factor does the temperature of the gas change if it expands adiabatically ( $\Delta S = 0$ ) from size  $L_1 = 2Na$  to size  $L_2 = 3Na$ ?

4.2 (This is a modified version of Problem 8.2 from Huang.)

Consider a classical system of  $N$  noninteracting diatomic molecules in a box of volume  $V$  at temperature  $T$ . The Hamiltonian for a single molecule is taken to be

$$H(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2) = \frac{1}{2m}(\vec{p}_1^2 + \vec{p}_2^2) + \frac{1}{2}K|\vec{r}_1 - \vec{r}_2|^2,$$

where  $\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2$  are the momenta and coordinates of the two atoms in a molecule.

- (a) Obtain the partition function for a single molecule,  $Q_1(T, V)$ .
- (b) Determine the partition function for  $N$  molecules, and obtain the Helmholtz free energy of the system.
- (c) Show that this gas obeys the ideal gas law  $PV = Nk_B T$ .
- (d) Determine the internal energy of this gas and show that your result is consistent with the equipartition theorem.
- (e) Find the specific heat at constant volume and the specific heat at constant pressure for this gas. Do you expect these results to describe a real diatomic gas at room temperature? If not, why not?
- (f) Find the mean square molecular diameter  $\langle |\vec{r}_1 - \vec{r}_2|^2 \rangle$ .

4.3 Problem 43, Chapter 3 of Pathria, Pg. 88-89.