Physics 5524

Statistical Mechanics

Problem Set 8

Due: Wednesday, Mar. 19

8.1 (From the Fall 2000 Comprehensive Exam).

Consider an ideal monatomic gas of N_1 particles of mass m confined to a volume V at a temperature T.

(a) Within the classical approximation and assuming the particles are indistinguishable, obtain an expression for the partition function for this gas.

(b) Determine the chemical potential μ of this gas.

A gas of N_2 of the same particles is adsorbed on a surface of area A, forming a two-dimensional ideal gas at temperature T on the surface. The energy of an adsorbed particle is $E = |\vec{p}|^2/2m - \eta$, where $\vec{p} = (p_x, p_y)$ is the two-dimensional momentum vector of the particle and η is the surface binding energy per particle.

(c) Using the same approximations as in (a) and (b), obtain an expression for the chemical potential μ of the adsorbed gas.

Now assume that the particles on the surface and in the surrounding three-dimensional gas are in equilibrium.

(d) Obtain an expression for the number of particles per unit area adsorbed on the surface ($n = N_2/A$) when the pressure of the surrounding three-dimensional gas is P.

8.2 Consider a two-dimensional gas of N noninteracting bosons with non-relativistic energy dispersion $\epsilon(\vec{k}) = \frac{\hbar^2 \vec{k}^2}{2m}$. (Here $\vec{k} = (k_x, k_y)$ is a two-dimensional wave vector). The gas has area A and is at temperature T.

(a) Find the one-particle density of states $a(\mathcal{E})$ for these bosons and show that it is equal to a constant for $\mathcal{E} > 0$, and 0 for $\mathcal{E} < 0$.

(b) Show that for this gas

$$\frac{N}{A} = \frac{z}{1-z}\frac{1}{A} + \frac{2\pi m k_B T}{h^2}\int_0^\infty dy \frac{1}{z^{-1}e^y - 1} \quad (1)$$

where the first term on the right hand side is N_0/A, the number of bosons per unit area in the condensate, and the second term on the right hand side is N_e/A, the number of bosons per unit area in excited states.

(c) The integral in Eq. (1) can easily be done analytically. Do it and show that this gas does *not* exhibit Bose condensation except at $T = 0$.

(d) Obtain an exact expression for the fugacity z of this gas as a function of $T > 0$, A and N. Show in the classical limit ($z \ll 1$) that $z = (\lambda/l)^2$ where $l = (A/N)^{1/2}$ is the mean particle spacing in two dimensions and λ is the thermal wavelength.

8.3 (From the Fall 2006 Qualifying Exam: Please try to resist looking up the solution!) A system of N bosons in two dimensions has an energy-momentum relationship $E = cp^{3/2}$ and number density $n = N/A$ (A is the area).

(a) Show that at low temperatures the system will Bose condense (unlike the 2D Bose gas in Problem 8.2), and that the Bose condensation temperature $T_c \sim n^{\alpha}$. Find α.

(b) Show that the entropy below T_C goes like $S \sim T^{\beta}$, and the "pressure" (which in two dimensions has units of force per unit length) goes as $P \sim T^{\gamma}$. Find β and γ.