**Physics 5524**

**Statistical Mechanics**

**Problem Set 9**

Due: Wednesday, Mar. 26

9.1 Consider a $D$-dimensional ideal Bose gas with single particle energy spectrum $\mathcal{E}(\vec{k}) = A|\vec{k}|^s$ where $s$ is some positive number. (**Note:** In doing this problem you should not have to evaluate an integrals explicitly.)

(a) Determine the conditions on $D$ and $s$ for which this gas exhibits Bose condensation with $T_c > 0$.

(b) For the cases when Bose condensation occurs, obtain an expression for the condensate fraction, $N_0/N$, where $N$ is the total number of particles and $N_0$ is the number of particles in the condensate, as a function of $T$ for $T \leq T_c$

(c) Again for the cases when Bose condensation occurs, show that when $T < T_c$ the pressure $P$ of this gas is proportional to $T^\alpha$ and independent of the density, and determine the exponent $\alpha$.

9.2 (From the Fall 1998 Comprehensive Exam)

An ideal Bose gas contained in a box of fixed volume $V$ consists of $N$ noninteracting bosons of mass $M$ each of which possesses an internal degree of freedom which can be described by assuming that the bosons are "two-level" systems. Bosons (with a fixed momentum $\vec{p}$) in the ground state have energy $E_0 = \frac{p^2}{2M}$, while bosons in the excited state have energy $E_1 = \frac{p^2}{2M} + \Delta$, where $\Delta > 0$ is the excitation energy. Assume that $\Delta \gg k_B T$.

(a) Compute the Bose-Einstein condensation temperature $T_c$ for this gas of two-level bosons.

(b) Obtain an expression for the amount by which the condensation temperature is raised or lowered due to the existence of the internal degree of freedom.

(c) For temperatures below $T_c$, obtain an expression for the condensate fraction of bosons in their ground state which occupy the zero-momentum state.

9.3 Pathria, Problem 7.8, Page 189.