Statistical Mechanics — PHY 5524

Midterm March 2, 2009

1. (60 pts) Consider a system of N noninteracting spin-1 nuclei placed in a magnetic field B at temperature T. The nuclei form a solid so you may assume that the spins are localized (i.e. distinguishable). The Hamiltonian for each spin is

$$H = -\mu B s,$$

where μ is the magnetic moment strength and $s = S_Z/\hbar$ can take the values -1, 0, and 1.

(a) Write down the partition function for this system and obtain the Helmholtz free energy.

(b) Determine the magnetization of this system $M = N\mu\langle s\rangle$ as a function of N, B and T.

(c) Consider the limit $k_B T \gg \mu B$ and obtain an approximate expression for M, valid to first order in $\frac{\mu B}{k_B T}$. Using this expression determine the magnetic susceptibility of the nuclei,

$$\chi(T) = \lim_{B\to 0}\left(\frac{\partial M}{\partial B}\right)_{N,T}.$$

(d) Now determine the entropy of this system. From your result show that for adiabatic processes B and T obey a law of the form $T^\alpha B = \text{Const.}$ and determine the exponent α.

2. (40 pts) Consider a classical gas consisting of N noninteracting particles confined to a volume V at temperature T. Assume the partition function for a single particle in this gas has the form

$$Q_1 = V f(T),$$

for some (unspecified) function $f(T)$.

(a) Write down the partition function for this gas of N particles and obtain an expression for the Helmholtz free energy A. Comment on the significance of the Gibbs factor in determining whether or not your expression for A is extensive.

(b) From the Helmholtz free energy determine the pressure P of this gas and show that it satisfies the ideal gas law regardless of the form of $f(T)$.

(c) Now assume that $f(T) = \gamma T^n$ for some constant γ and n. Obtain the energy of the gas E, and from this determine C_V, the specific heat at constant volume, and C_P, the specific heat at constant pressure.