Physics 5524

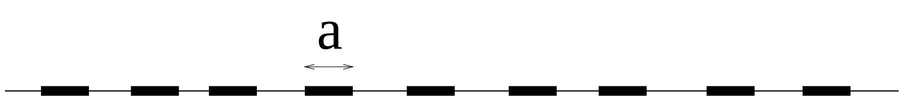
Statistical Mechanics

Problem Set 4

Due: Friday, Feb. 6

4.1 (This is a modified version of a problem from the Fall 1997 Comprehensive Exam.)

Consider a classical one dimensional gas of impenetrable rods of mass m and length a. The rods cannot overlap, but otherwise do not interact. Let L denote the length of the system and N denote the number of rods.



(a) Show that in the limit $L \gg Na$ the partition function of this gas is approximately given by

$$Q_N(T, L) \simeq \frac{(L - Na)^N}{N!} \left(\frac{2\pi m k_B T}{h^2} \right)^{N/2} . \qquad (1)$$

(b) Assuming the above approximate expression for Q_N is valid, calculate the Helmholtz free energy of this gas as a function of T and L.

(c) Find the equation of state for this gas relating the pressure P, the length L, and the temperature T. (In one dimension the pressure is simply the force the gas exerts on either end of the system).

(d) Determine the entropy for this gas as a function of T and L.

(e) How much work is done by this gas if it expands isothermally from size $L_1 = 2Na$ to size $L_2 = 3Na$?

(**Note:** Strictly speaking in this part (and part (f)) we are not in the limit $L \gg Na$, nonetheless, make the assumption that the approximations leading to (1) are still valid.)

(f) By what factor does the temperature of the gas change if it expands adiabatically ($\Delta S = 0$) from size $L_1 = 2Na$ to size $L_2 = 3Na$?

4.2 (This is a modified version of Problem 8.2 from Huang.)

Consider a classical system of N noninteracting diatomic molecules in a box of volume V at temperature T. The Hamiltonian for a single molecule is taken to be

$$H(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2) = \frac{1}{2m}(\vec{p}_1^{\,2} + \vec{p}_2^{\,2}) + \frac{1}{2}K|\vec{r}_1 - \vec{r}_2|^2,$$

where $\vec{p}_1$, $\vec{p}_2$, $\vec{r}_1$, $\vec{r}_2$ are the momenta and coordinates of the two atoms in a molecule.

(a) Obtain the partition function for a single molecule, $Q_1(T, V)$.

(b) Determine the partition function for N molecules, and obtain the Helmholtz free energy of the system.

(c) Show that this gas obeys the ideal gas law $PV = Nk_BT$.

(d) Determine the internal energy of this gas and show that your result is consistent with the equipartition theorem.

(e) Find the specific heat at constant volume and the specific heat at constant pressure for this gas. Do you expect these results to describe a real diatomic gas at room temperature? If not, why not?

(f) Find the mean square molecular diameter $\langle|\vec{r}_1 - \vec{r}_2|^2\rangle$.

4.3 Problem 43, Chapter 3 of Pathria, Pg. 88-89.