Physics 5524

Statistical Mechanics

Problem Set 9

Due: Monday, Mar. 30

9.1 (From the Fall 2006 Qualifying Exam)

A system of N bosons in two dimensions has an energy-momentum relationship $E = cp^{3/2}$ and number density $n = N/A$ (A is the area).

(a) Show that at low temperatures the system will Bose condense (unlike the 2D Bose gas in Problem 8.2), and that the Bose condensation temperature $T_c \sim n^\alpha$. Find α.

(b) Show that the entropy below T_C goes like $S \sim T^\beta$, and the "pressure" (which in two dimensions has units of force per unit length) goes as $P \sim T^\gamma$. Find β and γ.

9.2 Consider a D-dimensional ideal Bose gas with single particle energy spectrum $\mathcal{E}(\vec{k}) = A|\vec{k}|^s$ where s is some positive number. (**Note:** In doing this problem you should not have to evaluate an integrals explicitly.)

(a) Determine the conditions on D and s for which this gas exhibits Bose condensation with $T_c > 0$.

(b) For the cases when Bose condensation occurs, obtain an expression for the condensate fraction, N_0/N, where N is the total number of particles and N_0 is the number of particles in the condensate, as a function of T for $T \leq T_c$

(c) Again for the cases when Bose condensation occurs, show that when $T < T_c$ the pressure P of this gas is proportional to T^α and independent of the density, and determine the exponent α.

9.3 (From the Fall 1998 Comprehensive Exam)

An ideal Bose gas contained in a box of fixed volume V consists of N noninteracting bosons of mass M each of which possesses an internal degree of freedom which can be described by assuming that the bosons are "two-level" systems. Bosons (with a fixed momentum $\vec{p}$) in the ground state have energy $E_0 = \frac{p^2}{2M}$, while bosons in the excited state have energy $E_1 = \frac{p^2}{2M} + \Delta$, where $\Delta > 0$ is the excitation energy. Assume that $\Delta \gg k_B T$.

(a) Compute the Bose-Einstein condensation temperature T_c for this gas of two-level bosons.

(b) Obtain an expression for the amount by which the condensation temperature is raised or lowered due to the existence of the internal degree of freedom.

(c) For temperatures below T_c, obtain an expression for the condensate fraction of bosons in their ground state which occupy the zero-momentum state.

9.4 Pathria, Problem 7.8, Page 189.