PHZ 3113, Homework Set No. 13. Due Friday, April 17, 2009.

Problem 1

a. Show that the infinite series $\sum_{n=0}^{\infty} x^n$ is uniformly and absolutely convergent for all $|x| \leq R < 1$.

Hint: First use Weierstrass' M test to show that the series is uniformly and absolutely convergent for all $|x| \leq R$ if $\sum_{n=0}^{\infty} R^n$ is convergent. Then use a convergence test of your choice to establish the convergence of $\sum_{n=0}^{\infty} R^n$.

b. Find an explicit expression for $S(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ for all $|x| \leq R < 1$. State the theorems you use to justify your manipulations.

Problem 2

a. Show that the infinite series $S(x) = \sum_{n=1}^{\infty} nx^n$ is uniformly and absolutely convergent for all $|x| \leq R < 1$.

Hint: First use Weierstrass' M test to show that $S(x)$ is uniformly and absolutely convergent for all $|x| \leq R$ if $S(R) = \sum_{n=1}^{\infty} nR^n$ is convergent. Then use a convergence test of your choice to establish the convergence of $S(R)$.

b. Find an explicit expression for $S(x)$ for all $|x| \leq R < 1$. State the theorems you use to justify your manipulations.

Problems from W&A Ch. 5:
W&A 5.6.1
W&A 5.6.7
W&A 5.7.1

Also: Resume of the topics covered on sequences and series (parts of Ch. 5 plus Fourier series).